Name ______________________ ______ Review

Pre-Calculus Period( ) Polars

1) Specify 3 additional pairs of polar coordinates, two with $r > 0$, one with $r < 0$. Then plot the points on polar grid paper.

   a. $(-2, 210^\circ)$ b. $(4, \frac{3\pi}{2})$

2) Find the Cartesian coordinates of the point.

   a. $(5, 300^\circ)$ b. $(-2, \frac{\pi}{6})$

3) Name the quadrant in which the point is located. Then find two pairs of polar coordinates for each pair of Cartesian coordinates, one with $r > 0$ and one with $r < 0$ and $0^\circ \le \theta < 360^\circ$

   a. $(-5, 5)$ b. $(0, -7)$

4) Transform the polar equation into a Cartesian equation.

   a. $r = 6\cos\theta$ b. $r = \dfrac{6}{\cos\theta - 4\sin\theta}$

   c. $r = 4$ d. $r = \dfrac{-3}{\cos\theta}$

   e. $r\csc\theta = 6$ f. $\theta = -\dfrac{2\pi}{3}$

5) (NEW!) Transform the Cartesian equation into a polar equation. Isolate the r in your final answer.

   a. $x^2 + y^2 = 36$ b. $y = x + 1$

   c. $4x + 3y = 7$

6) Identify the graph of each polar equation and give <u>specific</u> identifying features of each. Then graph each equation on polar grid paper, showing all necessary work and/or tables of values.

   a. $r = -5$ b. $r = 4 + 5\cos\theta$

   c. $r\sin\theta = -6$ d. $r = \dfrac{5}{3\cos\theta - \sin\theta}$

   e. $r = -6\cos\theta$ f. $r = 3 - 3\sin\theta$

   g. $r = 5 + 2\cos\theta$ h. $r = 3\cos 2\theta$

Polars Review

1. a) $(-2, 210°)$

$\boxed{(-2, 570°)}$

$\boxed{(2, 390°)}$

$\boxed{(2, 30°)}$

b. $(4, \frac{3\pi}{2})$

$\boxed{(4, \frac{7\pi}{2})}$

$\boxed{(4, \frac{-\pi}{2})}$

$\boxed{(-4, \frac{\pi}{2})}$

2. a) $(5, 300°) \rightarrow \boxed{(\frac{5}{2}, \frac{-5\sqrt{3}}{2})}$

$x = 5\cos 300°$ $\quad$ $y = 5\sin 300°$

$x = 5(\frac{1}{2})$ $\quad$ $y = 5(\frac{-\sqrt{3}}{2})$

$x = \frac{5}{2}$ $\quad$ $y = \frac{-5\sqrt{3}}{2}$

b) $(-2, \frac{\pi}{6}) \rightarrow \boxed{(-\sqrt{3}, -1)}$

$x = -2\cos\frac{\pi}{6}$ $\quad$ $y = -2\sin\frac{\pi}{6}$

$x = -2(\frac{\sqrt{3}}{2})$ $\quad$ $y = -2(\frac{1}{2})$

$x = -\sqrt{3}$ $\quad$ $y = -1$

3. a) $(-5, 5)$ $\boxed{Q\,II}$

$r = \pm\sqrt{(-5)^2 + 5^2}$ $\quad$ $\tan\theta = \frac{5}{-5}$

$r = \pm\sqrt{25 + 25}$ $\quad$ $\tan\theta = -1$

$r = \pm\sqrt{50}$ $\quad$ $\theta = 135°, 315°$

$r = \pm 5\sqrt{2}$

$\boxed{(5\sqrt{2}, 135°)}$

$\boxed{(-5\sqrt{2}, 315°)}$

b) $(0, -7)$ $\boxed{y\text{-axis}}$

$r = \pm\sqrt{0^2 + (-7)^2}$ $\quad$ $\tan\theta = \frac{-7}{0}$

$r = \pm\sqrt{49}$ $\quad$ $\theta = 90°, 270°$

$r = \pm 7$

$\boxed{(7, 270°)}$

$\boxed{(-7, 90°)}$

4a) $r = 6\cos\theta$

$r = 6 \cdot \frac{x}{r}$

$r^2 = 6x$

$\boxed{x^2 + y^2 = 6x}$

$x^2 - 6x + y^2 = 0$

$(x^2 - 6x + 9) + y^2 = 9$

$\boxed{(x-3)^2 + y^2 = 9}$

4b) $r = \frac{6}{\cos\theta - 4\sin\theta}$

$r(\cos\theta - 4\sin\theta) = 6$

$r\cos\theta - 4r\sin\theta = 6$

$x - 4y = 6$

$x = 6 + 4y$

$x - 6 = 4y$

$\boxed{\frac{1}{4}x - \frac{3}{2} = y}$

4. c) $r = 4$

$\pm\sqrt{x^2 + y^2} = 4$

$\boxed{x^2 + y^2 = 16}$

4. d) $r = \frac{-3}{\cos\theta}$

$r\cos\theta = -3$

$\boxed{x = -3}$

4e) $r\csc\theta = 6$

$\frac{r}{\sin\theta} = 6$

$r = 6\sin\theta$

$r = 6 \cdot \frac{y}{r}$

$r^2 = 6y$

$\boxed{x^2 + y^2 = 6y}$

~~$x^2 + y^2 - 6y = 0$~~

~~$x^2 + (y^2 - 6y + 9) = 0 + 9$~~

~~$\boxed{x^2 + (y-3)^2 = 9}$~~

4f) $\theta = \frac{-2\pi}{3}$

$\tan^{-1}\frac{y}{x} = \frac{-2\pi}{3}$

$\frac{y}{x} = \tan\frac{-2\pi}{3}$

$\frac{y}{x} = \sqrt{3}$

$\boxed{y = \sqrt{3}x}$

$\tan\theta = y/x$

$\tan(-2\pi/3) = y/x$

$\sqrt{3} = y/x$

$y = \sqrt{3}x$

5. a) $x^2 + y^2 = 36$

$r^2 = 36$

$\boxed{r = \pm 6}$

5b) $y = x + 1$

$r\sin\theta = r\cos\theta + 1$

$r\sin\theta - r\cos\theta = 1$

$r(\sin\theta - \cos\theta) = 1$

$\boxed{r = \frac{1}{\sin\theta - \cos\theta}}$

5c) $4x + 3y = 7$

$4r\cos\theta + 3r\sin\theta = 7$

$r(4\cos\theta + 3\sin\theta) = 7$

$\boxed{r = \frac{7}{4\cos\theta + 3\sin\theta}}$

6.a) $r = -5$

circle centered at pole

radius = 5

6.b) $r = 4 + 5\cos\theta$

limacon symmetric across x-axis

has a loop

0°	$4 + 5\cos 0° = 4 + 5(1) = 9$
30°	$4 + 5\cos 30° = 4 + 5\left(\frac{\sqrt{3}}{2}\right) = 4 + \frac{5\sqrt{3}}{2}$
60°	$4 + 5\cos 60° = 4 + 5\left(\frac{1}{2}\right) = 6.5$
90°	$4 + 5\cos 90° = 4 + 5(0) = 4$
120°	$4 + 5\cos 120° = 4 + 5\left(-\frac{1}{2}\right) = 1.5$
150°	$4 + 5\cos 150° = 4 + 5\left(-\frac{\sqrt{3}}{2}\right) = 4 - \frac{5\sqrt{3}}{2}$
180°	$4 + 5\cos 180° = 4 + 5(-1) = -1$

6.c) $r\sin\theta = -6$

horizontal line at $y = -6$

30°	$\frac{-6}{\sin 30°} = \frac{-6}{1/2} = -12$
90°	$\frac{-6}{\sin 90°} = \frac{-6}{1} = -6$

6d) $r = \frac{5}{3\cos\theta - \sin\theta}$

oblique line, not through pole

0°	$\frac{5}{3\cos 0° - \sin 0°} = \frac{5}{3}$
90°	$\frac{5}{3\cos 90° - \sin 90°} = \frac{5}{-1} = -5$

6e) $r = -6\cos\theta$

circle with center off pole

radius = $\left|\frac{-6}{2}\right| = 3$

symmetric across x-axis

$a = -6 < 0$, circle to left of y-axis

0°	$-6\cos 0° = -6$
30°	$-6\cos 30° = -6\left(\frac{\sqrt{3}}{2}\right) = -3\sqrt{3}$
60°	$-6\cos 60° = -6\left(\frac{1}{2}\right) = -3$
90°	$-6\cos 90° = 0$

6f) $r = 3 - 3\sin\theta$

cardioid symmetric across y-axis

90°	$3 - 3\sin 90° = 3 - 3(1) = 0$
120°	$3 - 3\sin 120° = 3 - 3\left(\frac{\sqrt{3}}{2}\right) = 3 -$
150°	$3 - 3\sin 150° = 3 - 3\left(\frac{1}{2}\right) = 1.5$
180°	$3 - 3\sin 180° = 3 - 0 = 3$
210°	$3 - 3\sin 210° = 3 - 3\left(-\frac{1}{2}\right) = 4.$
240°	$3 - 3\sin 240° = 3 - 3\left(-\frac{\sqrt{3}}{2}\right) = 3$
270°	$3 - 3\sin 270° = 3 - 3(-1) = 6$

6.g) $r = 5 + 2\cos\theta$

limacon symmetric across x-axis, dimpled

$0°$	$5+2\cos 0° = 5+2(1) = 7$
$30°$	$5+2\cos 30° = 5+2\left(\frac{\sqrt{3}}{2}\right) = 5+\sqrt{3}$
$60°$	$5+2\cos 60° = 5+2\left(\frac{1}{2}\right) = 6$
$90°$	$5+2\cos 90° = 5$
$120°$	$5+2\cos 120° = 5+2\left(\frac{-1}{2}\right) = 4$
$150°$	$5+2\cos 150° = 5+2\left(\frac{-\sqrt{3}}{2}\right) = 5-\sqrt{3}$
$180°$	$5+2\cos 180° = 5+2(-1) = 3$

6h) $r = 3\cos 2\theta$

rose with 4 petals of length 3

$2\theta = 0° + 360°k$

$\theta = 0° + 180°k$

$k=0 \quad 0° + 180°(0) = 0° \quad (3, 0°)$

$k=1 \quad 0° + 180°(1) = 180° \quad (3, 180°)$

$2\theta = 180° + 360°k$

$\theta = 90° + 180°k$

$k=0 \quad 90° + 180°(0) = 90° \quad (3, 90°)$

$k=1 \quad 90° + 180°(1) = 270° \quad (3, 270°)$

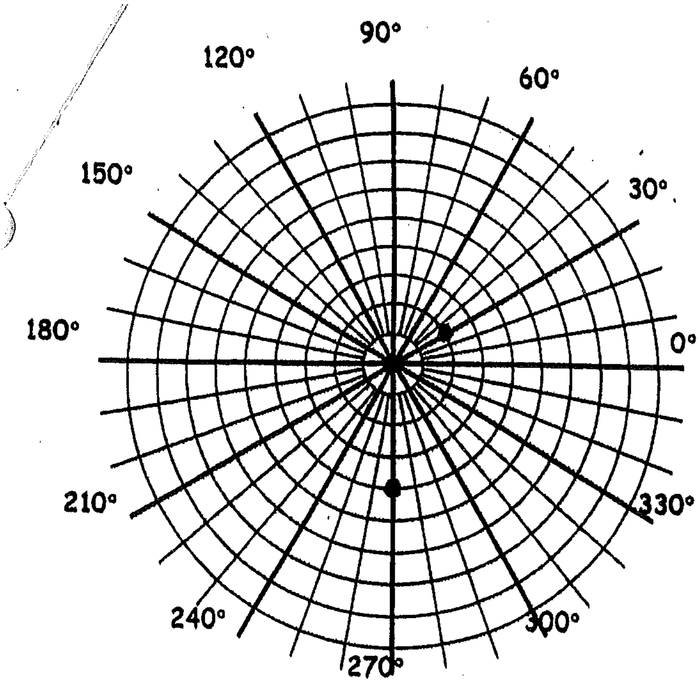
$r = 3\cos(2(15°))$ $\quad$ $r = 3\cos(2(30°))$

$r = 3\cos(30°)$ $\quad$ $r = 3\cos 60°$

$r = 3\left(\frac{\sqrt{3}}{2}\right)$ $\quad$ $r = 3\left(\frac{1}{2}\right)$

$r = \frac{3\sqrt{3}}{2}$ $\quad$ $r = \frac{3}{2}$

$\left(\frac{3\sqrt{3}}{2}, 15°\right)$ $\quad$ $\left(\frac{3}{2}, 30°\right)$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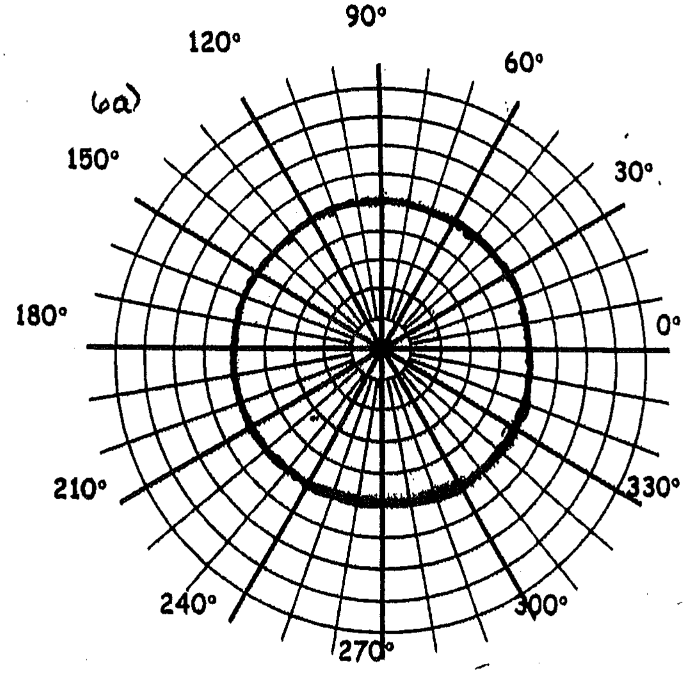
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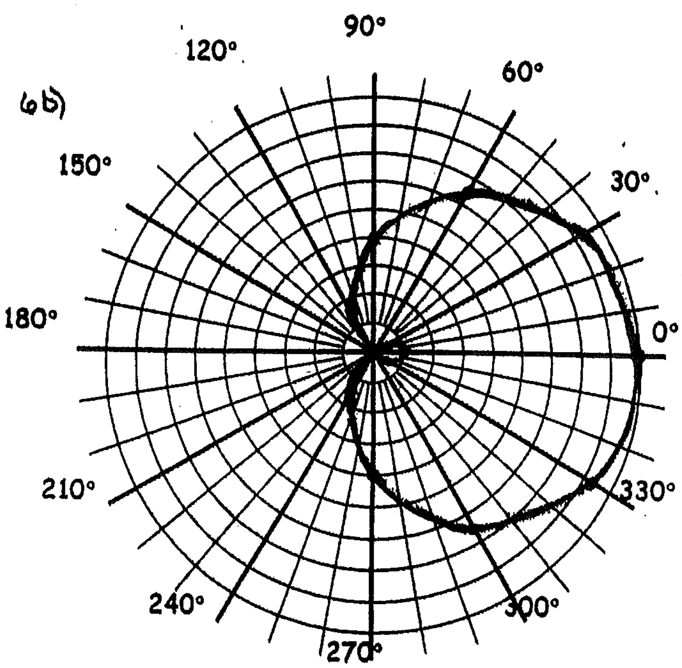
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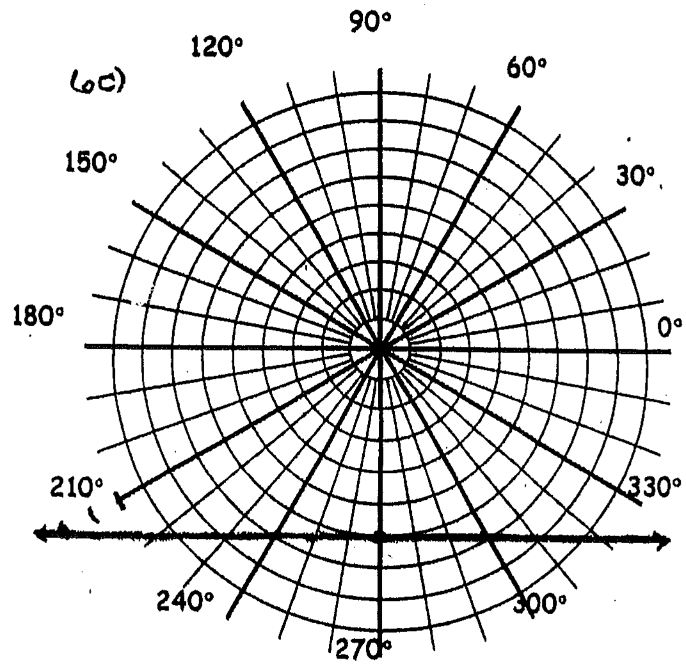
6c)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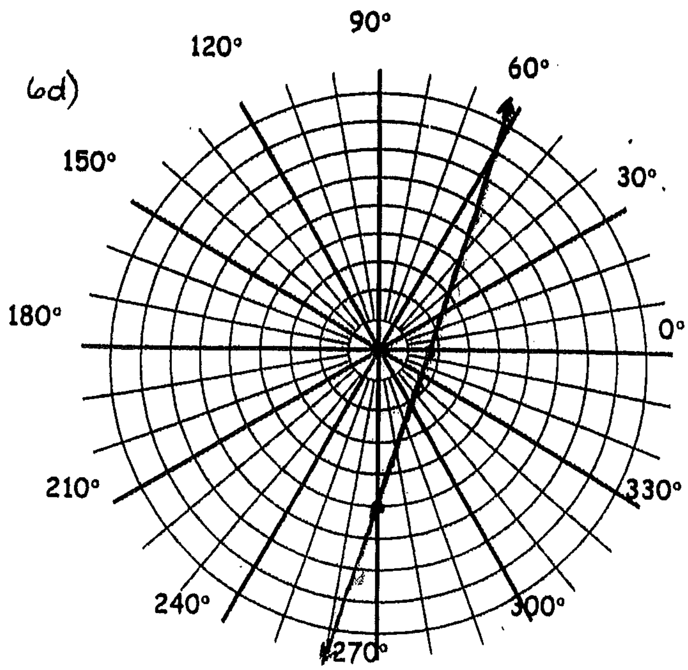
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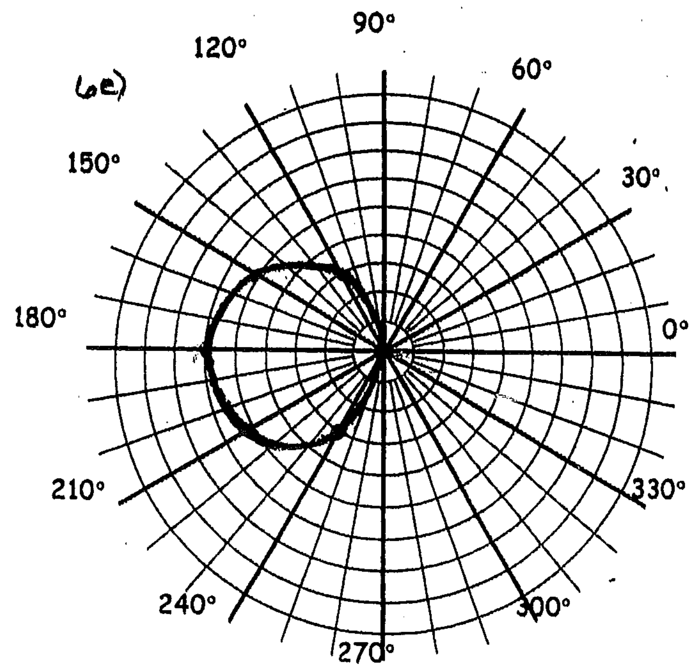
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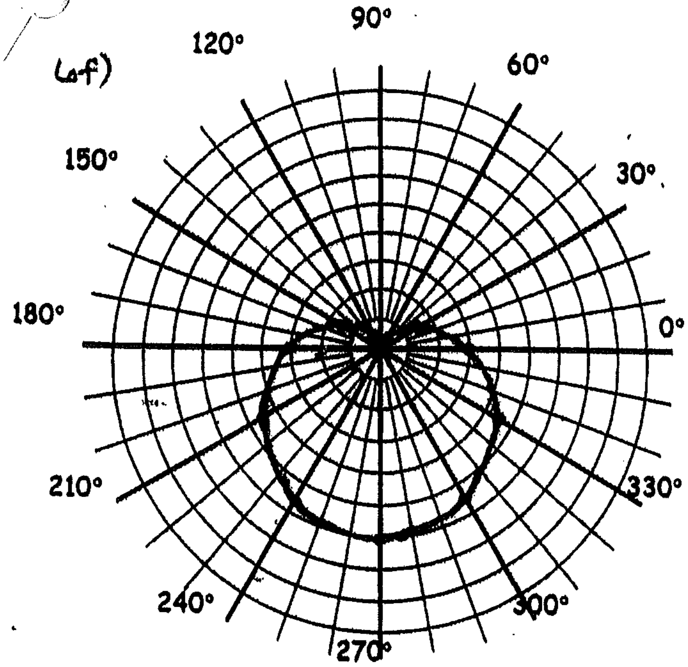

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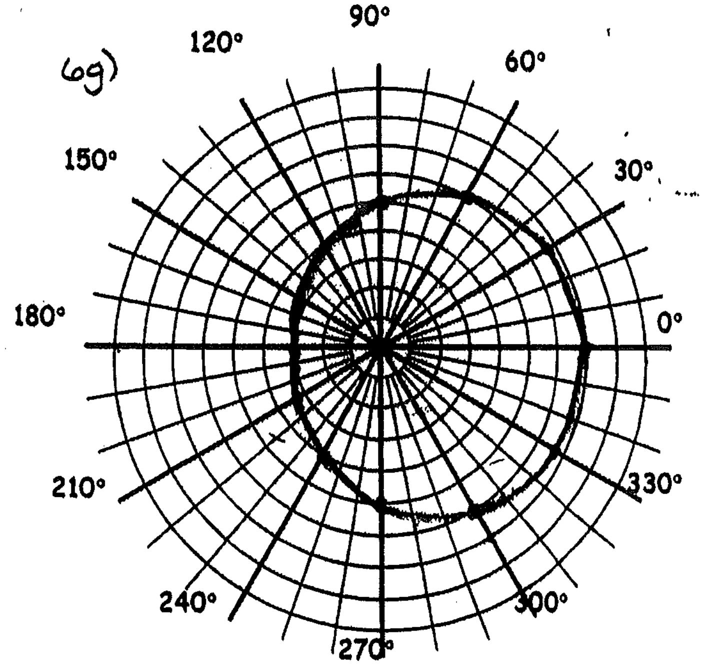

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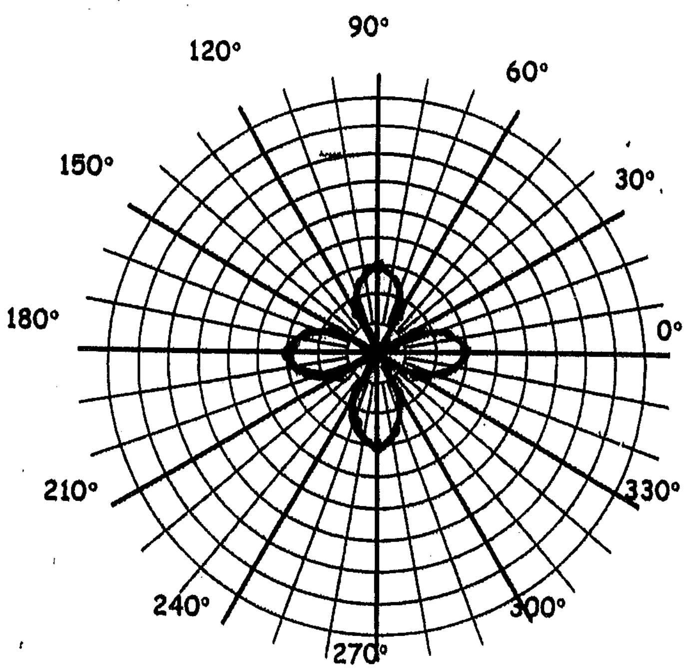

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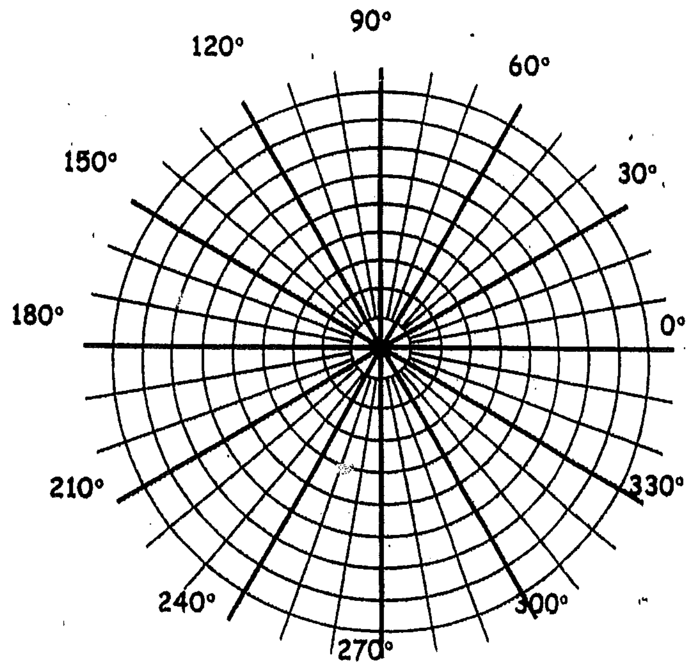

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