# Analyzing Functions

## Functions and Their Properties

# Properties to Analyze

- domain and range
- continuity
- increasing/decreasing
- boundedness
- intercepts
- local and absolute extrema
- symmetry
- asymptotes
- end behavior

# Domain and Range

- the domain of a function is all of the *possible* x-values
- usually the domain is all reals, but values that make the denominator = 0 or a square root of a negative must be excluded
- other excluded values will arise as we study more complicated functions
- the domain of a model is only the values that fit the situation

# Domain and Range

- the range is all the resulting y-values
- the easiest way to find the range is to examine the graph of the function
- if you cannot graph it, then try plugging in different types of domain values to see what types of outputs are created

# Continuity

- a graph is continuous if it can be sketched without lifting your pencil
- graphs become discontinuous 3 ways
  - holes (removable discontinuity)
  - jumps (jump discontinuity)
  - vertical asymptotes (infinite discontinuity)
- continuity can relate to the whole function, an interval, or a particular point

# Increasing/Decreasing

- at a particular point, a function can be increasing, decreasing, or constant
- it is easiest to figure out by looking at a graph of the function
- if the graph has positive slope at the point then it is increasing, negative slope then it is decreasing, and slope=0 then it is constant
- we use interval notation to describe where a function is increasing, decreasing, or constant

# Boundedness

- if a graph has a minimum value then we say that it is bounded below
- if a graph has a maximum value then we say that it is bounded above
- a graph that has neither a min or max is said to be unbounded
- a graph that has both a min and max is said to be bounded

# Local and Absolute Extrema

- if a graph changes from increasing to decreasing and vice versa, then it has peaks and valleys
- the point at the tip of a peak is called a local max
- the point at the bottom of a valley is called a local min
- if the point is the maximum value of the whole function then it can also be called an absolute max (similar for absolute min)

# Symmetry

- graphs that look the same on the left side of the y-axis as the right side are said to have y-axis symmetry
- if a graph has y-axis symmetry then the function is even
- the algebraic test for y-axis sym. is to plug $-x$ into the function, and it can be simplified back into the original function

$$f(-x) = f(x)$$

# Symmetry

- graphs that look the same above the x-axis as below are said to have x-axis symmetry
- the algebraic test for x-axis sym. is to plug $-y$ into the equation, and it can be simplified back into the original equation
- graphs with x-axis symmetry are usually not a function

# Symmetry

- graphs that can be rotated 180° and still look the same are said to have symmetry with respect to the origin
- if a graph has origin symmetry then the function is called odd
- the algebraic test for origin symmetry is to plug $-x$ into the function, and it can be simplified back into the opposite of the original function $f(-x) = -f(x)$

# Asymptotes

- some curves appear to flatten out to the right and to the left, the imaginary line they approach is called a horizontal asymptote
- some curves appear to approach an imaginary vertical line as they approach a specific value (they have a steep climb or dive), these are called vertical asymptotes
- we use dashed lines for an asymptote

# Limit Notation

- a new type of notation used to describe the behavior of curves near asymptotes (and for many other things as well)
- $\lim\limits_{x \to a} f(x)$ represents the value that the function $f(x)$ approaches as $x$ gets closer to $a$

# Asymptote Notation

- if y = $b$ is a horizontal asymptote of $f(x)$, then

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to \infty} f(x) = b$$

- if x = $a$ is a vertical asymptote of $f(x)$, then

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a+} f(x) = \pm\infty$$

# End Behavior

- end behavior is a description of the nature of the curve for very large and small x-values (as x approaches $\pm\infty$)
- a horizontal asymptote gives one type of end behavior
- if the f(x) values continue to increase (or decrease) as the x values approach $\pm\infty$ then a limit can be used to describe the behavior

  ex. If $f(x) = x^2$ then $\lim_{x \to \pm\infty} f(x) = \infty$