SECTION 10.3 EXERCISES

In Exercises 1–10, find the limit by direct substitution if it exists.

1. $\lim_{x \to -1} x(x-1)^2$

2. $\lim_{x \to 3} (x-1)^{12}$

3. $\lim_{x \to 2} (x^3 - 2x + 3)$

4. $\lim_{x \to -2} (x^3 - x + 5)$

5. $\lim_{x \to 2} \sqrt{x+5}$

6. $\lim_{x \to -2} (x-4)^{2/3}$

7. $\lim_{x \to 0} (e^x \sin x)$

8. $\lim_{x \to \pi} \ln\left(\sin \frac{x}{2}\right)$

9. $\lim_{x \to a} (x^2 - 2)$

10. $\lim_{x \to a} \frac{x^2 - 1}{x^2 + 1}$

In Exercises 11–18, **(a)** explain why you cannot use substitution to find the limit and **(b)** find the limit algebraically if it exists.

11. $\lim_{x \to -3} \frac{x^2 + 7x + 12}{x^2 - 9}$

12. $\lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 15}$

13. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

14. $\lim_{x \to 2} \frac{x^3 - 2x^2 + x - 2}{x - 2}$

15. $\lim_{x \to -2} \frac{x^2 - 4}{x + 2}$

16. $\lim_{x \to -2} \frac{|x^2 - 4|}{x + 2}$

17. $\lim_{x \to 0} \sqrt{x - 3}$

18. $\lim_{x \to 0} \frac{x - 2}{x^2}$

In Exercises 19–22, use the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$, along with the limit properties, to find the following limits.

19. $\lim_{x \to 0} \frac{\sin x}{2x^2 - x}$

20. $\lim_{x \to 0} \frac{\sin 3x}{x}$

21. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

22. $\lim_{x \to 0} \frac{x + \sin x}{2x}$

In Exercises 23–26, find the limits.

23. $\lim_{x \to 0} \frac{e^x - \sqrt{x}}{\log_4(x + 2)}$

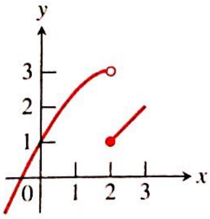
24. $\lim_{x \to 0} \frac{3\sin x - 4\cos x}{5\sin x + \cos x}$

25. $\lim_{x \to \pi/2} \frac{\ln(2x)}{\sin^2 x}$

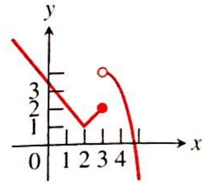
26. $\lim_{x \to 27} \frac{\sqrt{x + 9}}{\log_3 x}$

In Exercises 27–30, use the given graph to find the limits or to explain why the limits do not exist.

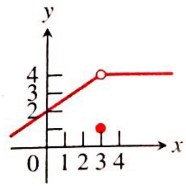
27. **(a)** $\lim_{x \to 2^-} f(x)$
(b) $\lim_{x \to 2^+} f(x)$
(c) $\lim_{x \to 2} f(x)$



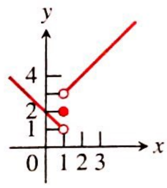
28. **(a)** $\lim_{x \to 3^-} f(x)$
(b) $\lim_{x \to 3^+} f(x)$
(c) $\lim_{x \to 3} f(x)$



29. **(a)** $\lim_{x \to 3^-} f(x)$
(b) $\lim_{x \to 3^+} f(x)$
(c) $\lim_{x \to 3} f(x)$

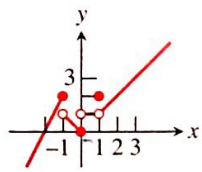


30. **(a)** $\lim_{x \to 1^-} f(x)$
(b) $\lim_{x \to 1^+} f(x)$
(c) $\lim_{x \to 1} f(x)$

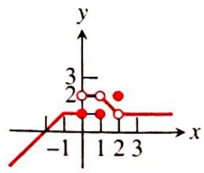


In Exercises 31 and 32, the graph of a function $y = f(x)$ is given. Which of the statements about the function are true and which are false?

31. **(a)** $\lim_{x \to -1^+} f(x) = 1$
(b) $\lim_{x \to 0^-} f(x) = 0$
(c) $\lim_{x \to 0^-} f(x) = 1$
(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$
(e) $\lim_{x \to 0} f(x)$ exists
(f) $\lim_{x \to 0} f(x) = 0$
(g) $\lim_{x \to 0} f(x) = 1$
(h) $\lim_{x \to 1} f(x) = 1$
(i) $\lim_{x \to 1} f(x) = 0$
(j) $\lim_{x \to 2^-} f(x) = 2$



32. **(a)** $\lim_{x \to -1^+} f(x) = 1$
(b) $\lim_{x \to 2} f(x)$ does not exist
(c) $\lim_{x \to 2} f(x) = 2$
(d) $\lim_{x \to 1^-} f(x) = 2$
(e) $\lim_{x \to 1^+} f(x) = 1$
(f) $\lim_{x \to 1} f(x)$ does not exist
(g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$
(h) $\lim_{x \to c} f(x)$ exists for every c in $(-1, 1)$.
(i) $\lim_{x \to c} f(x)$ exists for every c in $(1, 3)$.



In Exercises 33 and 34, use a graph of f to find **(a)** $\lim_{x\to 0^-} f(x)$, **(b)** $\lim_{x\to 0^+} f(x)$, and **(c)** $\lim_{x\to 0} f(x)$ if they exist.

33. $f(x) = (1+x)^{1/x}$ **34.** $f(x) = (1+x)^{1/(2x)}$

35. Group Activity Assume that $\lim_{x\to 4} f(x) = -1$ and $\lim_{x\to 4} g(x) = 4$. Find the limit.

(a) $\lim_{x\to 4} (g(x) + 2)$ **(b)** $\lim_{x\to 4} xf(x)$

(c) $\lim_{x\to 4} g^2(x)$ **(d)** $\lim_{x\to 4} \dfrac{g(x)}{f(x) - 1}$

36. Group Activity Assume that $\lim_{x\to a} f(x) = 2$ and $\lim_{x\to a} g(x) = -3$. Find the limit.

(a) $\lim_{x\to a} (f(x) + g(x))$ **(b)** $\lim_{x\to a} (f(x) \cdot g(x))$

(c) $\lim_{x\to a} (3g(x) + 1)$ **(d)** $\lim_{x\to a} \dfrac{f(x)}{g(x)}$

In Exercises 37–40, complete the following for the given piecewise-defined function f.

(a) Draw the graph of f.

(b) Determine $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

(c) Writing to Learn Does $\lim_{x\to a} f(x)$ exist? If it does, give its value. If it does not exist, give an explanation.

37. $a = 2, f(x) = \begin{cases} 2 - x & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ x^2 - 4 & \text{if } x > 2 \end{cases}$

38. $a = 1, f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ x + 1 & \text{if } x \ge 1 \end{cases}$

39. $a = 0, f(x) = \begin{cases} |x - 3| & \text{if } x < 0 \\ x^2 - 2x & \text{if } x \ge 0 \end{cases}$

40. $a = -3, f(x) = \begin{cases} 1 - x^2 & \text{if } x \ge -3 \\ 8 - x & \text{if } x < -3 \end{cases}$

In Exercises 41–46, find the limit.

41. $\lim_{x\to 2^+} \text{int}(x)$ **42.** $\lim_{x\to 2^-} \text{int}(x)$

43. $\lim_{x\to 0.0001} \text{int}(x)$ **44.** $\lim_{x\to 5/2^-} \text{int}(2x)$

45. $\lim_{x\to -3^+} \dfrac{x+3}{|x+3|}$ **46.** $\lim_{x\to 0^-} \dfrac{5x}{|2x|}$

In Exercises 47–54, find **(a)** $\lim_{x\to\infty} y$ and **(b)** $\lim_{x\to -\infty} y$.

47. $y = \dfrac{\cos x}{1 + x}$ **48.** $y = \dfrac{x + \sin x}{x}$

49. $y = 1 + 2^x$ **50.** $y = \dfrac{x}{1 + 2^x}$

51. $y = x + \sin x$ **52.** $y = e^{-x} + \sin x$

53. $y = -e^x \sin x$ **54.** $y = e^{-x}\cos x$

In Exercises 55–60, use graphs and tables to find the limit and identify any vertical asymptotes.

55. $\lim_{x\to 3^-} \dfrac{1}{x - 3}$ **56.** $\lim_{x\to 3^+} \dfrac{x}{x - 3}$

57. $\lim_{x\to -2^+} \dfrac{1}{x + 2}$ **58.** $\lim_{x\to -2^-} \dfrac{x}{x + 2}$

59. $\lim_{x\to 5} \dfrac{1}{(x - 5)^2}$ **60.** $\lim_{x\to 2} \dfrac{1}{x^2 - 4}$

In Exercises 61–64, determine the limit algebraically if possible. Support your answer graphically.

61. $\lim_{x\to 0} \dfrac{(1 + x)^3 - 1}{x}$ **62.** $\lim_{x\to 0} \dfrac{1/(3 + x) - 1/3}{x}$

63. $\lim_{x\to 0} \dfrac{\tan x}{x}$ **64.** $\lim_{x\to 2} \dfrac{x - 4}{x^2 - 4}$

In Exercises 65–72, find the limit.

65. $\lim_{x\to 0} \dfrac{|x|}{x^2}$ **66.** $\lim_{x\to 0} \dfrac{x^2}{|x|}$

67. $\lim_{x\to 0} \left[x \sin\left(\dfrac{1}{x}\right)\right]$ **68.** $\lim_{x\to 27} \cos\left(\dfrac{1}{x}\right)$

69. $\lim_{x\to 1} \dfrac{x^2 + 1}{x - 1}$ **70.** $\lim_{x\to\infty} \dfrac{\ln x^2}{\ln x}$

71. $\lim_{x\to\infty} \dfrac{\ln x}{\ln x^2}$ **72.** $\lim_{x\to\infty} 3^{-x}$

Standardized Test Questions

73. True or False If $f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 8 - x & \text{if } x > 3 \end{cases}$, then $\lim_{x\to 3} f(x)$ is undefined. Justify your answer.

74. True or False If $f(x)$ and $g(x)$ are two functions and $\lim_{x\to 0} f(x)$ does not exist, then $\lim_{x\to 0} [f(x) \cdot g(x)]$ cannot exist. Justify your answer.

Multiple Choice In Exercises 75–78, match the function $y = f(x)$ with the table. Do not use a calculator.

X	Y_1	
2.7	−52.3	
2.8	−82.2	
2.9	−172.1	
3	ERROR	
3.1	188.1	
3.2	98.2	
3.3	68.3	
X=2.7		

(a)

X	Y_1	
2.7	3.7	
2.8	3.8	
2.9	3.9	
3	ERROR	
3.1	4.1	
3.2	4.2	
3.3	4.3	
X=2.7		

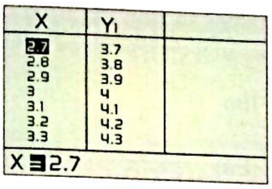
(b)

X	Y_1	
2.7	23.7	
2.8	33.8	
2.9	63.9	
3	ERROR	
3.1	−55.9	
3.2	−25.8	
3.3	−15.7	
X=2.7		

(c)

X	Y_1	
2.7	24.39	
2.8	25.24	
2.9	26.11	
3	ERROR	
3.1	27.91	
3.2	28.84	
3.3	29.79	
X=2.7		

(d)



(e)

75. $y = \dfrac{x^2 - 2x - 3}{x - 3}$

76. $y = \dfrac{x^2 + 2x + 3}{x - 3}$

77. $y = \dfrac{x^2 - 2x - 9}{x - 3}$

78. $y = \dfrac{x^3 - 27}{x - 3}$

Explorations

In Exercises 79–82, complete the following for the given piecewise-defined function f.

(a) Draw the graph of f.

(b) At what points c in the domain of f does $\lim_{x \to c} f(x)$ exist?

(c) At what points c does only the left-hand limit exist?

(d) At what points c does only the right-hand limit exist?

79. $f(x) = \begin{cases} \cos x & \text{if } -\pi \le x < 0 \\ -\cos x & \text{if } 0 \le x \le \pi \end{cases}$

80. $f(x) = \begin{cases} \sin x & \text{if } -\pi \le x < 0 \\ \csc x & \text{if } 0 \le x \le \pi \end{cases}$

81. $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{if } -1 \le x < 0 \\ x & \text{if } 0 \le x < 1 \\ 2 & \text{if } x = 1 \end{cases}$

82. $f(x) = \begin{cases} x^2 & \text{if } -2 \le x < 0 \text{ or } 0 < x \le 2 \\ 1 & \text{if } x = 0 \\ 2x & \text{if } x < -2 \text{ or } x > 2 \end{cases}$

83. Rabbit Population The population of rabbits over a 2-year period in a certain county is given in Table 10.5.

Table 10.5 Rabbit Population

Beginning of Month	Number (in thousands)
0	10
2	12
4	14
6	16
8	22
10	30
12	35
14	39
16	44
18	48
20	50
22	51

(a) Draw a scatter plot of the data in Table 10.5.

(b) Find a logistic regression model for the data. Find the limit of that model as time approaches infinity.

(c) What can you conclude about the limit of the rabbit population growth in the county?

(d) Provide a reasonable explanation for the population growth limit.

In Exercises 84–87, **(a)** make a table of function values for $x = 1, 0.1, 0.01, 0.001, 0.0001, 0.00001$ and 0.000001, **(b)** use the information in (a) to estimate $\lim_{x \to 0} f(x)$, if it exists, and **(c)** draw a graph to support your limit estimate in (b).

84. $f(x) = \dfrac{2^x - 1}{2^x}$

85. $f(x) = x \sin \dfrac{1}{x}$

86. $f(x) = \dfrac{1}{x} \sin \dfrac{1}{x}$

87. $f(x) = x^2 \sin(\ln |x|)$

In Exercises 88–91, sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

88. Group Activity $\lim_{x \to 0} f(x) = \infty$, $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = 2$

89. Group Activity $\lim_{x \to 4} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = 2$

90. Group Activity $\lim_{x \to \infty} f(x) = 2$, $\lim_{x \to -2^+} f(x) = -\infty$, $\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \infty$

91. Group Activity $\lim_{x \to 1} f(x) = \infty$, $\lim_{x \to 2^+} f(x) = -\infty$, $\lim_{x \to 2^-} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \infty$

Extending the Ideas

92. Properties of Limits Find the limits of f, g, and fg as x approaches c.

(a) $f(x) = \dfrac{2}{x^2}$, $g(x) = x^2$, $c = 0$

(b) $f(x) = \left|\dfrac{1}{x}\right|$, $g(x) = \sqrt[3]{x}$, $c = 0$

(c) $f(x) = \left|\dfrac{3}{x - 1}\right|$, $g(x) = (x - 1)^2$, $c = 1$

(d) $f(x) = \dfrac{1}{(x - 1)^4}$, $g(x) = (x - 1)^2$, $c = 1$

(e) Writing to Learn Suppose that $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = 0$. Based on your results in parts (a)–(d), what can you say about $\lim_{x \to a} (f(x) \cdot g(x))$?