

The Ellipse

Graph an ellipse and identify its important parts.

Write the equation of an ellipse in standard form given its important parts.

Identify an ellipse and write the equation in standard form given a conic equation in non standard form.

Standard Forms of an Ellipse

◆ The standard form of the equation of an ellipse with center at (h, k) .

Horizontal

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertices $(h \pm a, k)$

Co-Vertices $(h, k \pm b)$

$$\text{Foci: } c^2 = a^2 - b^2$$

Foci $(h \pm c, k)$

Vertical

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Vertices $(h, k \pm a)$

Co-Vertices $(h \pm b, k)$

Foci $(h, k \pm c)$

#1 Graph the ellipse

Label the Center Vertices, Co-Vertices, and Foci.

$$4x^2 + 25y^2 = 100$$

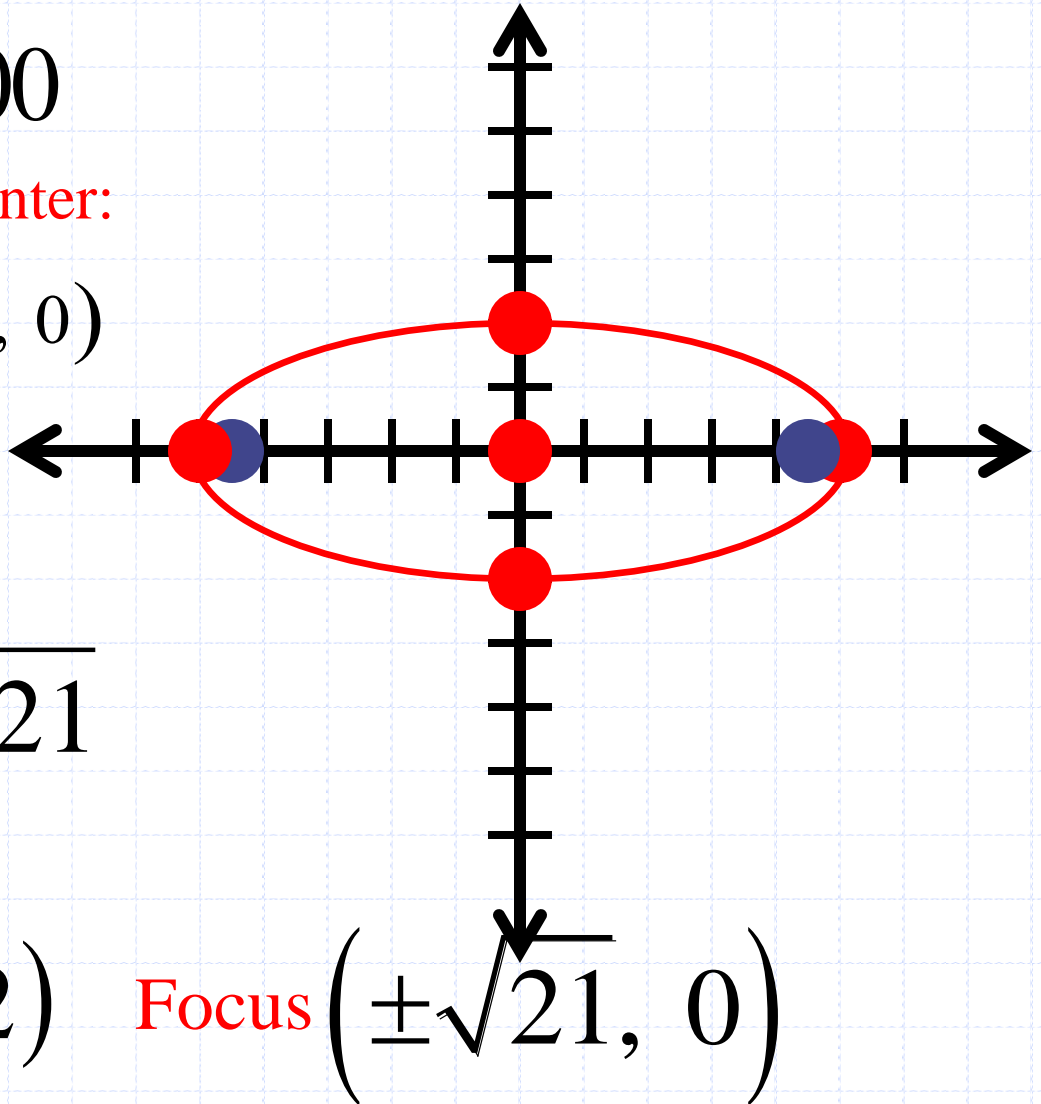
$$\frac{x^2}{25} + \frac{y^2}{4} = 1 \quad \text{Center: } (0, 0)$$

$$a = 5 \quad b = 2$$

$$c = \sqrt{25 - 4} = \sqrt{21}$$

$$\text{Vertices } (\pm 5, 0)$$

$$\text{Co-Vertices } (0, \pm 2) \quad \text{Focus } (\pm\sqrt{21}, 0)$$



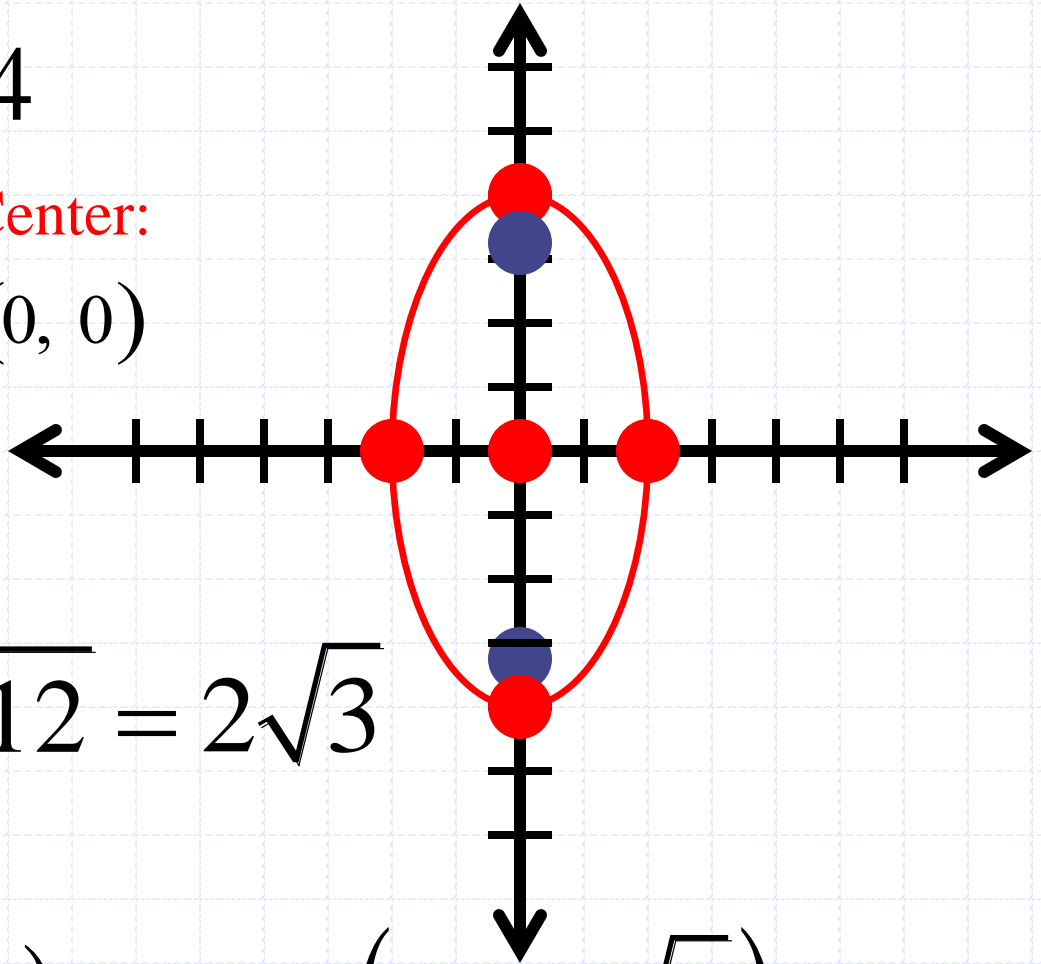
#2 Graph the ellipse

Label the Center, Vertices, Co-Vertices, and Foci.

$$16x^2 + 4y^2 = 64$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

Center:
(0, 0)



$$a = 4 \quad b = 2$$

$$c = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$$

Vertices $(0, \pm 4)$

Co-Vertices $(\pm 2, 0)$ Focus $(0, \pm 2\sqrt{3})$

#3 Graph the ellipse

Label the Center Vertices, Co-Vertices, and Foci.

$$\frac{(x-4)^2}{20} + \frac{(y+2)^2}{36} = 1$$

$$a = 6 \quad b = 2\sqrt{5}$$

$$c = \sqrt{36 - 20}$$

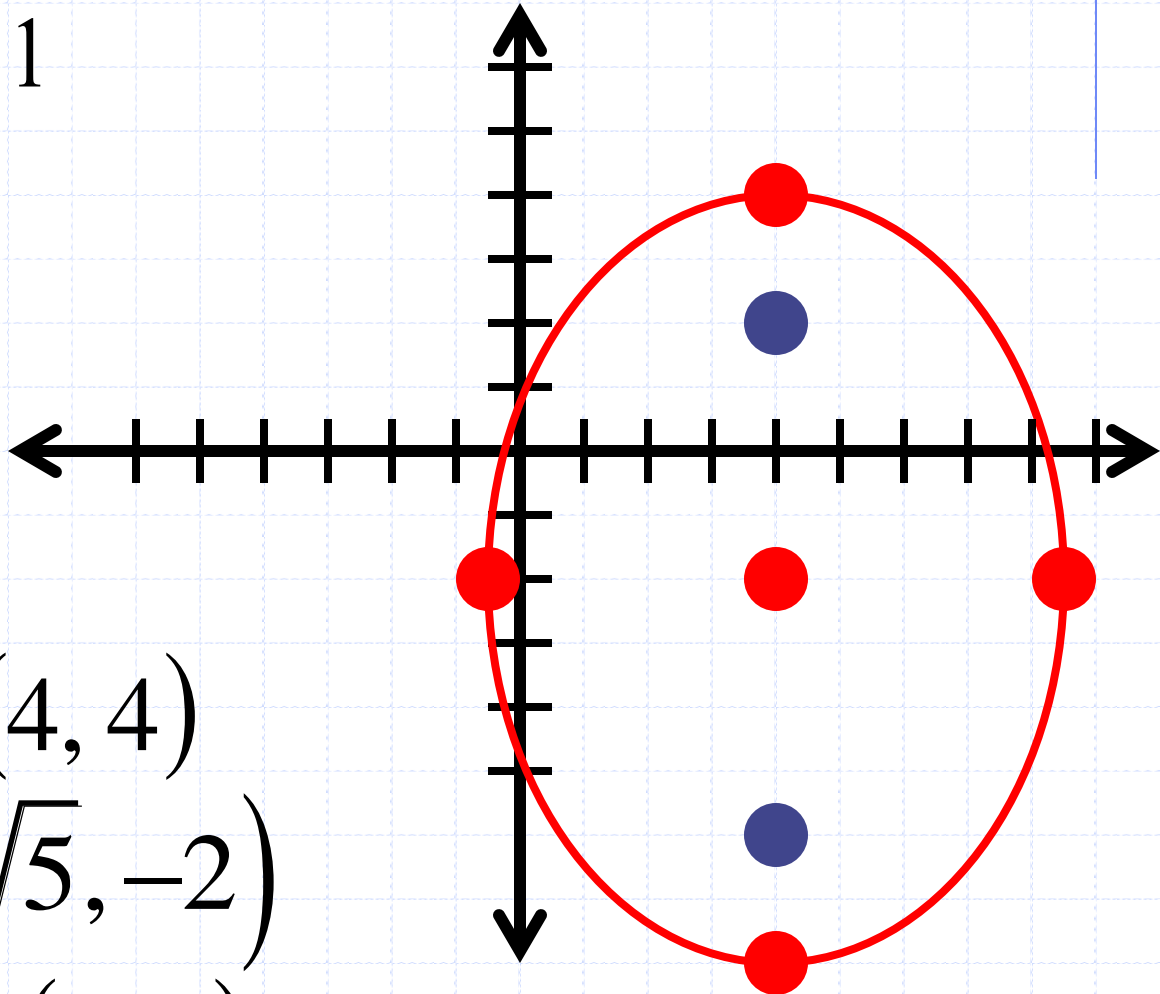
$$c = 4$$

Center $(4, -2)$

Vertices $(4, -8), (4, 4)$

Co-vertices $(4 \pm 2\sqrt{5}, -2)$

Foci $(4, -6), (4, 2)$



#4 Draw the ellipse

Label the Center, Vertices, Co-Vertices, and Foci.

$$\frac{(x+3)^2}{25} + \frac{(y-1)^2}{4} = 1$$

$$a = 5 \quad b = 2$$

$$c = \sqrt{25 - 4}$$

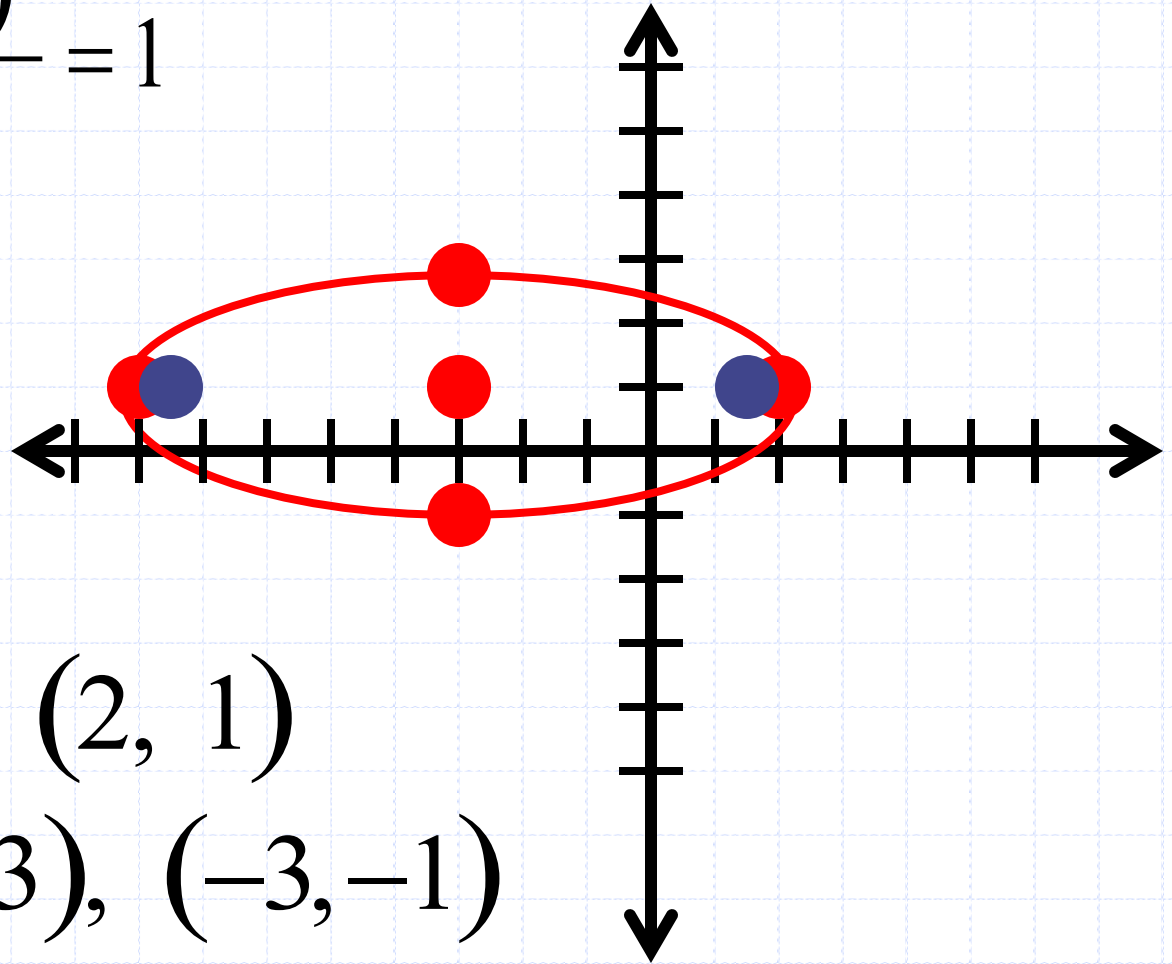
$$c = \sqrt{21} \approx 4.6$$

Center $(-3, 1)$

Vertices $(-8, 1), (2, 1)$

Co-vertices $(-3, 3), (-3, -1)$

Foci $(-3 \pm \sqrt{21}, 1)$



#5 Writing Equations of Ellipses

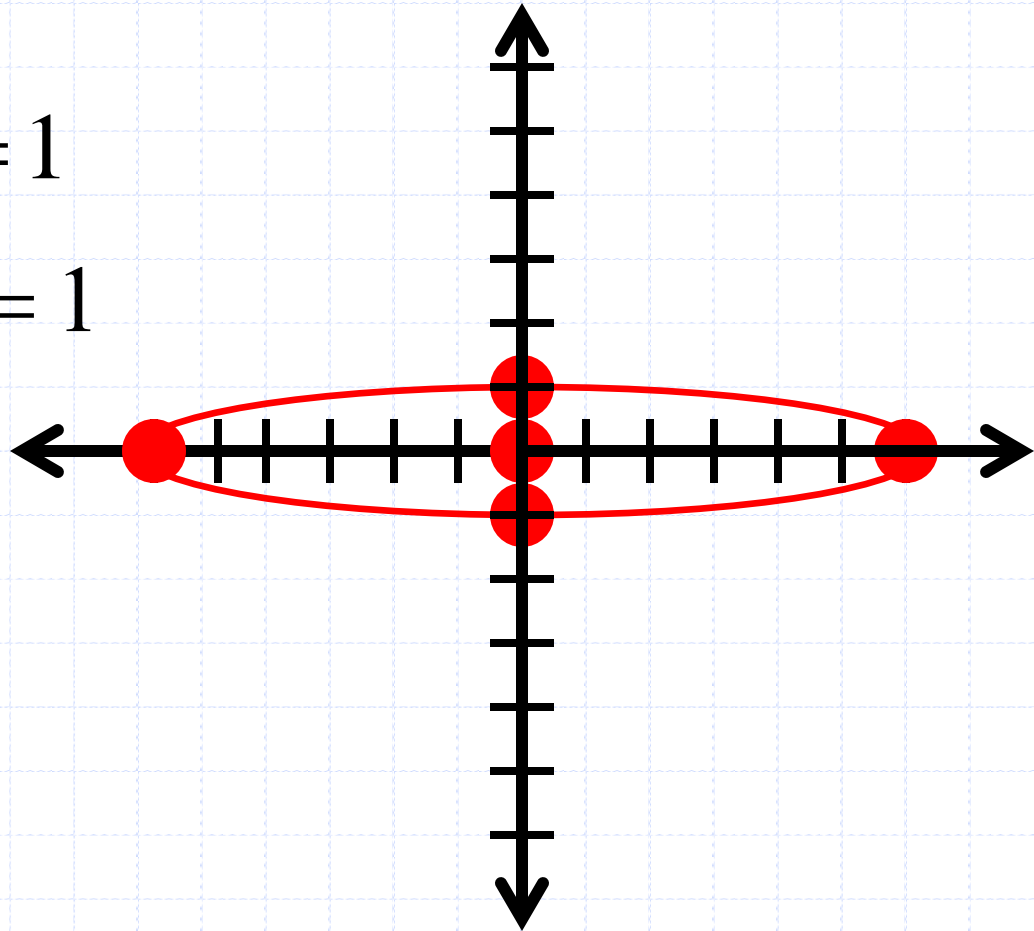
- ◆ Write an equation of the ellipse with the Vertex $(-6, 0)$, Co-Vertex $(0, -1)$, and Center $(0, 0)$.

Horizontal:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$h = 0 \quad k = 0 \quad a = 6 \quad b = 1$$

$$\frac{x^2}{36} + \frac{y^2}{1} = 1$$



#6 Writing Equations of Ellipses

◆ Write an equation of the ellipse with the

Center $(1, 4)$. Focus $(1, 4 + \sqrt{12})$, and Vertex $(1, 0)$,

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

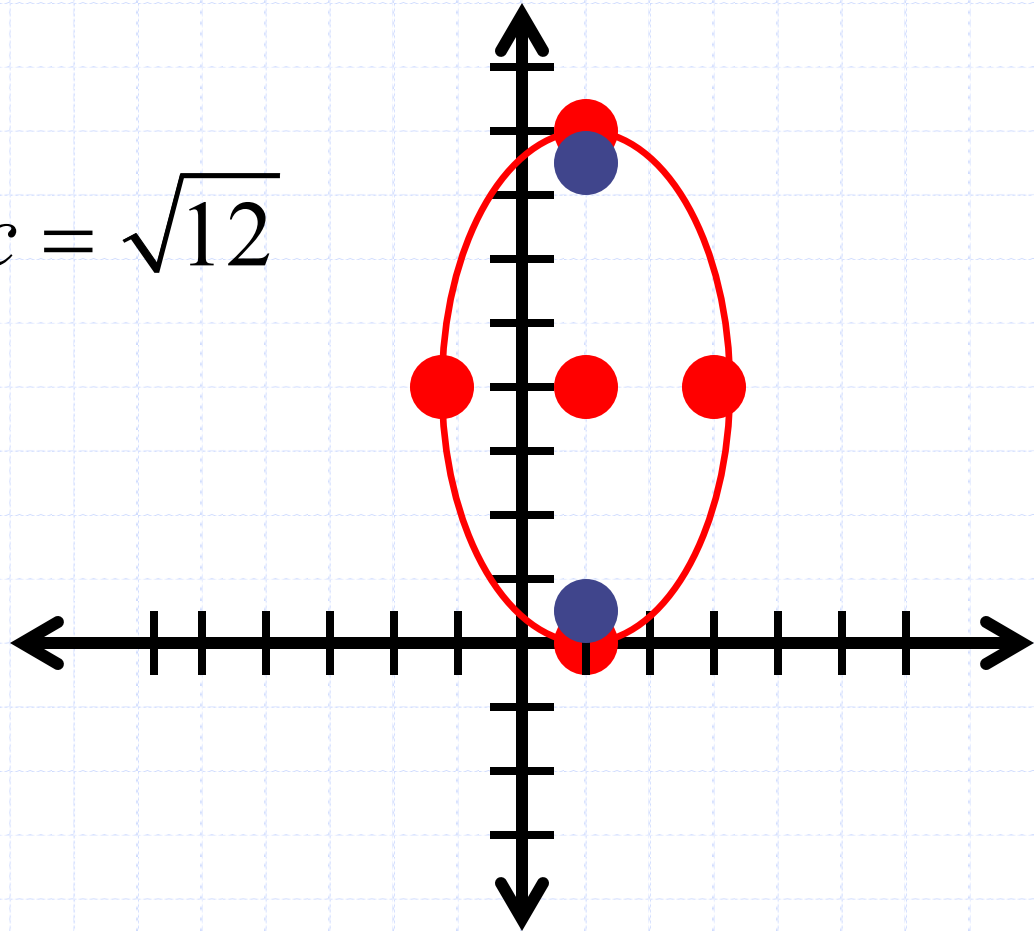
$$h = 1 \quad k = 4 \quad a = 4 \quad c = \sqrt{12}$$

$$(\sqrt{12})^2 = 4^2 - b^2$$

$$b^2 = 16 - 12$$

$$b = 2$$

$$\frac{(x-1)^2}{4} + \frac{(y-4)^2}{16} = 1$$



#7 Writing Equations of Ellipses

◆ Write an equation of the ellipse with the Vertex

○ $(-1, -2)$, Focus $(-1, -1)$, and Center $(-1, 3)$.

Vertical:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

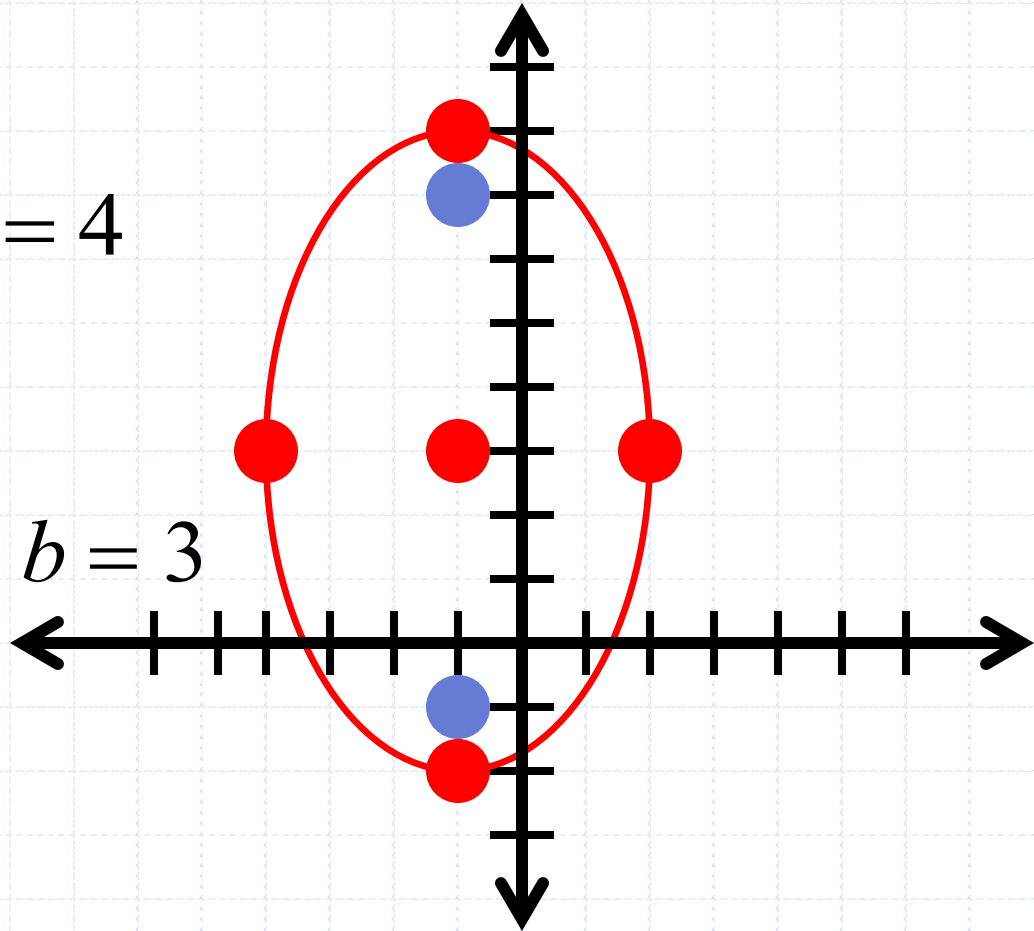
$$h = -1 \quad k = 3 \quad a = 5 \quad c = 4$$

$$c^2 = a^2 - b^2$$

$$4^2 = 5^2 - b^2$$

$$b^2 = 25 - 16 \quad b^2 = 9$$

$$\frac{(x+1)^2}{9} + \frac{(y-3)^2}{25} = 1$$



#8 Write the equation of the Ellipse in standard form.

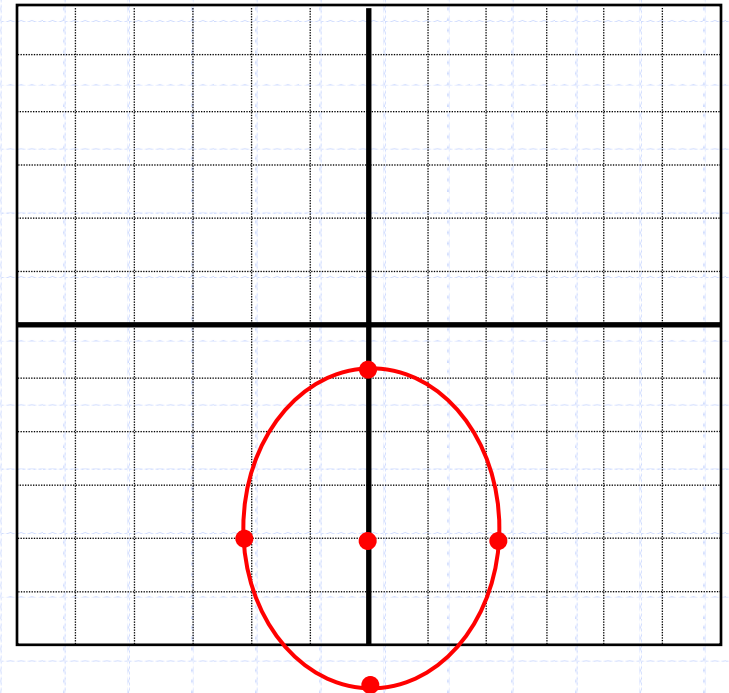
$$2x^2 + y^2 + 8y + 6 = 0$$

$$2x^2 + y^2 + 8y = -6$$

$$2x^2 + (y^2 + 8y + \underline{16}) = -6 + \underline{16}$$

$$2x^2 + (y + 4)^2 = 10$$

$$\frac{x^2}{5} + \frac{(y + 4)^2}{10} = 1$$



#9 Write the equation of the Ellipse in standard form.

$$x^2 + 4y^2 - 2x - 3 = 0$$

$$x^2 - 2x + 4y^2 = 3$$

$$x^2 - 2x + \underline{1} + 4y^2 = 3 + \underline{1}$$

$$(x - 1)^2 + 4y^2 = 4$$

$$\frac{(x - 1)^2}{4} + \frac{y^2}{1} = 1$$

