CHAPTER 7 REVIEW EXERCISES

1. (a) $\begin{bmatrix} 1 & 2 \\ 8 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} -3 & 4 \\ 0 & -3 \end{bmatrix}$ **(c)** $\begin{bmatrix} 2 & -6 \\ -8 & 0 \end{bmatrix}$ **(d)** $\begin{bmatrix} -7 & 11 \\ 4 & -6 \end{bmatrix}$ **3.** $\begin{bmatrix} -3 & -7 & 11 \\ 0 & -12 & 24 \end{bmatrix}$; not possible

5. $[3 \quad 7]$; not possible **7.** $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -3 & 4 \\ -2 & 1 & -1 \end{bmatrix}$; $\begin{bmatrix} -3 & 2 & 4 \\ 2 & 1 & -3 \\ 1 & -2 & -1 \end{bmatrix}$ **9.** $AB = BA = I_4$ **11.** $\begin{bmatrix} -2 & -5 & 6 & -1 \\ 0 & -1 & 1 & 0 \\ 10 & 24 & -27 & 4 \\ -3 & -7 & 8 & -1 \end{bmatrix}$

13. 20 **15.** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ **17.** $\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ **19.** $(1, 2)$ **21.** No solution **23.** $(-z - w + 2, w + 1, z, w)$

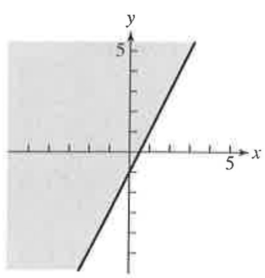
25. No solution **27.** $(-2z + w + 1, z - w + 2, z, w)$ **29.** $\left(\frac{9}{4}, -\frac{3}{4}, -\frac{7}{4}\right)$ **31.** No solution **33.** $(-w + 2, z + 3, z, w)$

35. $(-2, 1, 3, -1)$ **37.** $\approx(7.57, 42.71)$ **39.** $(x, y) \approx (0.14, -2.29)$ **41.** $(x, y) = (-2, 1)$ or $(x, y) = (2, 1)$

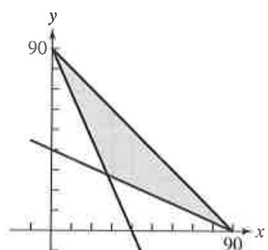
43. $(x, y) \approx (2.27, 1.53)$ **45.** $(a, b, c, d) = \left(\frac{17}{840}, -\frac{33}{280}, -\frac{571}{420}, \frac{386}{35}\right)$ **47.** $\frac{1}{x+1} + \frac{2}{x-4}$ **49.** $\frac{-1}{x+2} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$

51. $\frac{2}{x+1} + \frac{3x-4}{x^2+1}$ **53.** (c) **55.** (b)

57.

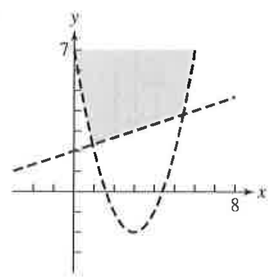


59.



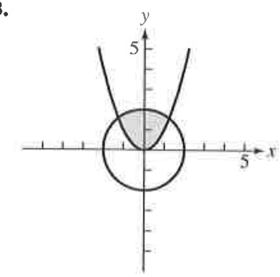
Corners at (0, 90), (90, 0), (360/13, 360/13). Boundaries included.

61.



Corners at $\approx$ (0.92, 2.31) and $\approx$ (5.41, 3.80). Boundaries excluded.

63.

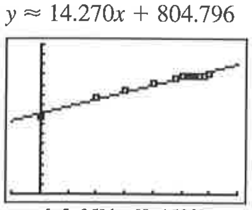


Corners at $\approx$ (−1.25, 1.56) and $\approx$ (1.25, 1.56). Boundaries included.

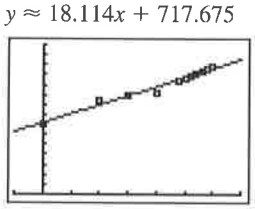
65. The minimum is 106 at (10, 6); there is no maximum. **67.** The minimum is 205 at (10, 25); the maximum is 292 at (4, 40).

69. (a) $\approx (2.12, 0.71)$ **(b)** $\approx (-0.71, 2.12)$

71. (a) $y \approx 14.270x + 804.796$ **(b)** $y \approx 18.114x + 717.675$ **(c)** In 1992



[–5, 35] by [0, 1500]



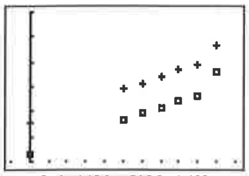
[–5, 35] by [0, 1500]

73 (a) $N = [200 \quad 400 \quad 600 \quad 250]$ **(b)** $P = [\$80 \quad \$120 \quad \$200 \quad \$300]$ **(c)** $NP^T = \$259{,}000$

75. 2 vans, 4 small trucks, 3 large trucks **77.** \$160,000 at 4%, \$170,000 at 6.5%, \$320,000 at 9%

79. Pipe A: 15 hours; Pipe B: $\approx$ 5.45 hours; Pipe C: 12 hours

Chapter 7 Project

1. 

[−1, 11] by [120, 150]

Males: $y \approx 1.515x + 120.729$;
Females: $y \approx 1.465x + 127.125$

3. Yes; no; no

5. Males: $y \approx \dfrac{412.574}{1 + 10.956e^{(-0.01539x)}}$; Females: $y \approx \dfrac{315.829}{1 + 9.031e^{(-0.01831x)}}$; (45, 64);
This represents the time when the female population became greater than the male population. (159, 212); This represents the time when the male population will again become greater than the female population.

7. Approx. 49.1% male and 50.9% female

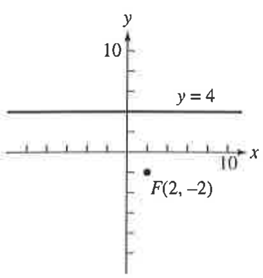
SECTION 8.1

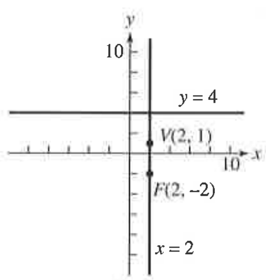
Exploration 1

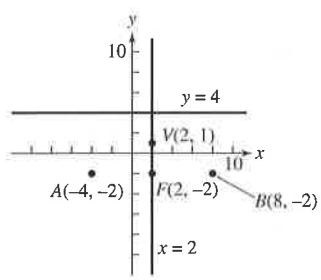
1. The axis of the parabola with focus (0, 1) and directrix $y = -1$ is the y-axis because it is perpendicular to $y = -1$, and passes through (0, 1). The vertex lies on this axis midway between the directrix and the focus, so is the point (0, 0).

3. $\{(-2\sqrt{6}, 6), (-2\sqrt{5}, 5), (-4, 4), (-2\sqrt{3}, 3), (-2\sqrt{2}, 2), (-2, 1), (0, 0), (2, 1), (2\sqrt{2}, 2), (2\sqrt{3}, 3), (4, 4), (2\sqrt{5}, 5), (2\sqrt{6}, 6)\}$

Exploration 2

1. 

3. 

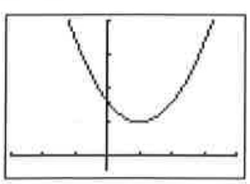
5. 

7. Downward

Quick Review 8.1

1. $\sqrt{13}$ 3. $y = \pm 2\sqrt{x}$ 5. $y + 6 = -(x - 1)^2$

7. Vertex: (1, 5); $f(x)$ can be obtained from $g(x)$ by stretching x^2 by 3, shifting up 5 units, and shifting right 1 unit.



[−3, 4] by [−2, 20]

9. $f(x) = -2(x + 1)^2 + 3$

Exercises 8.1

1. Vertex: (0, 0); Focus: $\left(0, \frac{3}{2}\right)$; Directrix: $y = -\frac{3}{2}$; Focal width: 6

3. Vertex: (−3, 2); Focus: (−2, 2); Directrix: $x = -4$; Focal width: 4

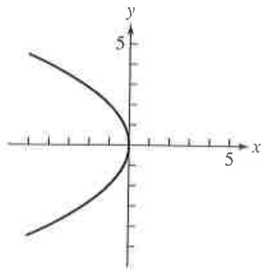
5. Vertex: (0, 0); Focus: $\left(0, -\frac{1}{3}\right)$; Directrix: $y = \frac{1}{3}$; Focal width: $\frac{4}{3}$

7. (c) 9. (a) 11. $y^2 = -12x$ 13. $x^2 = -16y$ 15. $x^2 = 20y$ 17. $y^2 = 8x$

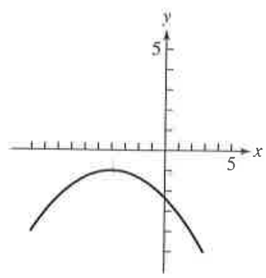
19. $x^2 = -6y$ 21. $(y + 4)^2 = 8(x + 4)$ 23. $(x - 3)^2 = 6\left(y - \frac{5}{2}\right)$ 25. $(y - 3)^2 = -8(x - 4)$

27. $(x-2)^2 = 16(y+1)$ **29.** $(y+4)^2 = -10(x+1)$

31.



33.



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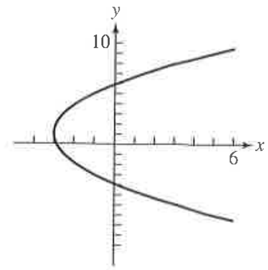

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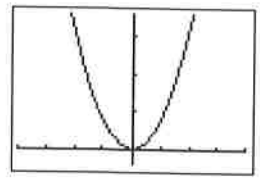
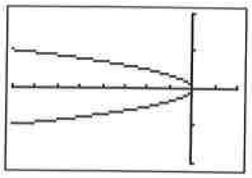


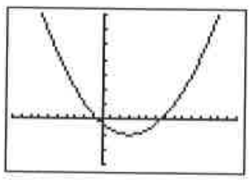
[−4, 4] by [−2, 18]

39.



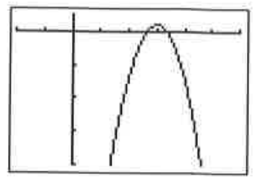
[−8, 2] by [−2, 2]

41.



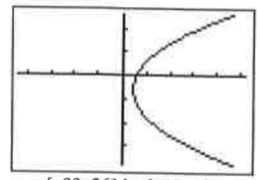
[−10, 15] by [−3, 7]

43.



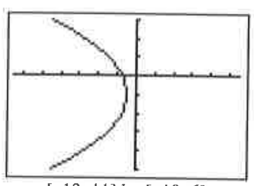
[−2, 6] by [−40, 15]

45.



[−22, 26] by [−19, 13]

47.



[−13, 11] by [−10, 6]

49. Completing the square, the equation becomes $(x+1)^2 = y-2$, a parabola with vertex $(-1, 2)$, focus $(-1, 9/4)$, and directrix $y = 7/4$.

51. Completing the square, the equation becomes $(y-2)^2 = 8(x-2)$, a parabola with vertex $(2, 2)$, focus $(4, 2)$, and directrix $x = 0$.

53. $(y-2)^2 = -6x$ **55.** $(x-2)^2 = -4(y+1)$

57. The derivation only requires that p is a fixed real number.

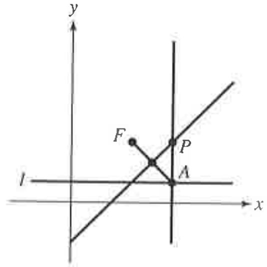
59. The filament should be placed 1.125 cm from the vertex along the axis of the mirror.

61. The electronic receiver is located 2.5 units from the vertex along the axis of the parabolic microphone.

63. Starting at the leftmost tower, the lengths of the cables are: $\approx$ {79.44, 54.44, 35, 21.11, 12.78, 10, 12.78, 21.11, 35, 54.44, 79.44}

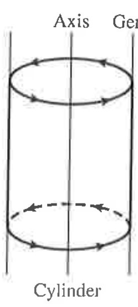
65. False. Every point on a parabola is the same distance from its focus and its directrix.

67. (d) **69.** (b) **71.** **(a)–(c)**

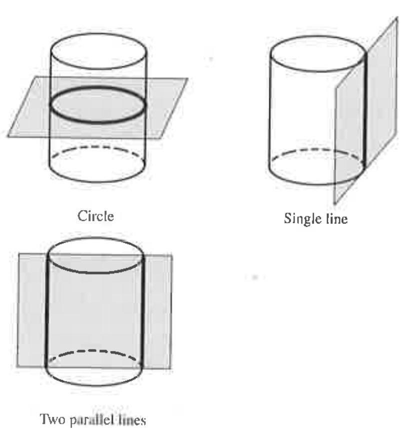


(d) a parabola

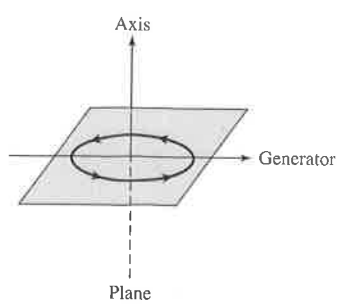
73. (a)



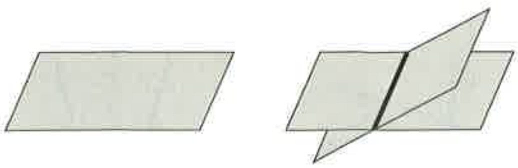
(b)



(c)



(d)



SECTION 8.2

Exploration 1

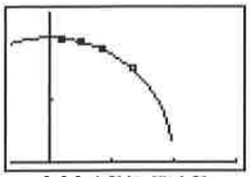
1. $x = -2 + 3 \cos t$ and $y = 5 + 7 \sin t$ $\cos t = \frac{x+2}{3}$ and $\sin t = \frac{y-5}{7}$; $\cos^2 t$ and $\sin^2 t = 1$ yields the equation $\frac{(x+2)^2}{9} + \frac{(y-5)^2}{49} = 1.$ **3.** Example 1: $x = 3 \cos t$ and $y = 2 \sin t$. Example 2: $x = 2 \cos t$ and $y = \sqrt{13} \sin t$. Example 3: $x = 5 \cos t + 3$, $y = 4 \sin t - 1$.

5. Example 1: $x = 3 \cos t$, $y = 2 \sin t$;
$\cos t = \frac{x}{3}$, $\sin t = \frac{y}{2}$;
$\cos^2 t + \sin^2 t = 1$ yields $\frac{x^2}{9} + \frac{y^2}{4} = 1$, or $4x^2 + 9y^2 = 36$.
Example 2: $x = 2 \cos t$ and $y = \sqrt{13} \sin t$;
$\cos t = \frac{x}{2}$, $\sin t = \frac{y}{\sqrt{13}}$;
$\cos^2 t + \sin^2 t = 1$ yields $\frac{y^2}{13} + \frac{x^2}{4} = 1$.
Example 3: $x = 3 + 5 \cos t$, $y = -1 + 4 \sin t$;
$\cos t = \frac{x-3}{5}$, $\sin t = \frac{y+1}{4}$;
$\cos^2 t + \sin^2 t = 1$ yields $\frac{(x-3)^2}{25} + \frac{(y+1)^2}{16} = 1$.

Exploration 2

3. $a = 8$ cm, $b \approx 7.75$ cm, $c = 2$ cm, $e \approx 0.25$, $b/a \approx 0.97$; $a = 7$ cm, $b \approx 6.32$ cm, $c = 3$ cm, $e \approx 0.43$, $b/a \approx 0.90$; $a = 6$ cm, $b \approx 4.47$ cm, $c = 4$ cm, $e \approx 0.67$, $b/a \approx 0.75$.

5.



[–0.3, 1.5] by [0, 1.2]

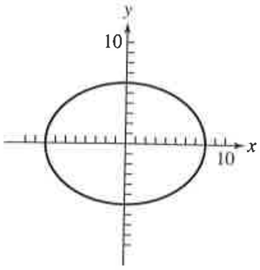
$b/a = \sqrt{1 - e^2}$

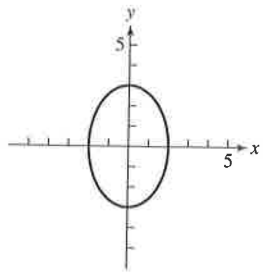
Quick Review 8.2

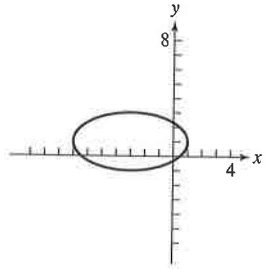
1. $\sqrt{61}$ **3.** $y = \pm\frac{3}{2}\sqrt{4 - x^2}$ **5.** $x = 8$ **7.** $x = 2, x = -2$ **9.** $x = \frac{3 \pm \sqrt{15}}{2}$

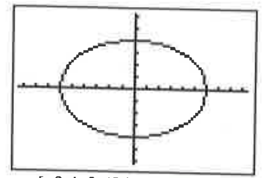
Exercises 8.2

1. Vertices: (4, 0), (−4, 0); Foci: (3, 0), (−3, 0) **3.** Vertices: (0, 6), (0, −6); Foci: (0, 3), (0, −3)
5. Vertices: (2, 0), (−2, 0); Foci: (1, 0), (−1, 0) **7.** (d) **9.** (a)

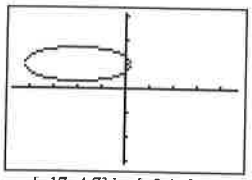
11. 

13. 

15. 

17. 

[−9.4, 9.4] by [−6.2, 6.2]

19. 

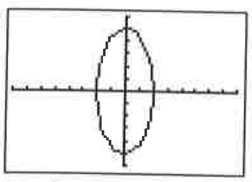
[−17, 4.7] by [−3.1, 3.1]

21. $\frac{y^2}{9} + \frac{x^2}{4} = 1$ **23.** $\frac{x^2}{25} + \frac{y^2}{21} = 1$ **25.** $\frac{y^2}{25} + \frac{x^2}{16} = 1$ **27.** $\frac{y^2}{36} + \frac{x^2}{16} = 1$

29. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ **31.** $\frac{(y-2)^2}{36} + \frac{(x-1)^2}{16} = 1$ **33.** $\frac{(x-3)^2}{9} + \frac{(y+4)^2}{5} = 1$

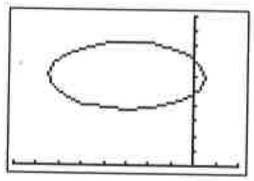
35. $\frac{(y+2)^2}{25} + \frac{(x-3)^2}{9} = 1$ **37.** Center: $(-1, 2)$; Vertices: $(-6, 2)$, $(4, 2)$; Foci: $(-4, 2)$, $(2, 2)$.

39. Center: $(7, -3)$; Vertices: $(7, 6)$, $(7, -12)$; Foci: $(7, -3 \pm \sqrt{17})$

41. 

[−8, 8] by [−6, 6]

$x = 2 \cos t,\ y = 5 \sin t$

43. 

[−8, 2] by [0, 10]

$x = 2\sqrt{3} \cos t - 3,\ y = \sqrt{5} \sin t + 6$

45. Vertices: $(1, -4)$, $(1, 2)$; Foci: $(1, -1 \pm \sqrt{5})$; Eccentricity: $\frac{\sqrt{5}}{3}$

47. Vertices: $(-7, 1)$, $(1, 1)$; Foci: $(-3 \pm \sqrt{7}, 1)$; Eccentricity: $\frac{\sqrt{7}}{4}$

49. $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$ **53.** $a = 237{,}086.5$, $b \approx 236{,}571$, $c = 15{,}623.5$, $e \approx 0.066$ **55.** ≈ 1347 Gm, ≈ 1507 Gm

57. For sungrazers, $a - c < 1.5(1.392) = 2.088$. **59.** $(\pm\sqrt{51.75}, 0) \approx (\pm 7.19, 0)$ **61.** $(-2, 0)$, $(2, 0)$

63. **(a)** Approximate solutions: $(\pm 1.04, -0.86)$, $(\pm 1.37, 0.73)$

(b) $\left(\pm\frac{\sqrt{94 - 2\sqrt{161}}}{8}, -\frac{1+\sqrt{161}}{16}\right)$, $\left(\pm\frac{\sqrt{94 + 2\sqrt{161}}}{8}, \frac{-1+\sqrt{161}}{16}\right)$

65. False. The distance is $a(1-e)$. **67.** (c) **69.** (b)

71. **(a)** When $a = b = r$, $A = \pi ab = \pi rr = \pi r^2$ and $P \approx \pi(2r) \cdot \left(3 - \sqrt{(4r)(4r)}\,/\,(2r)\right) = \pi(2r) \cdot (3 - 2) = 2\pi r$.

(b) Answers will vary.

SECTION 8.3

Exploration 1

1. $x = -1 + 3/\cos t = -1 + 3 \sec t$ and $y = 1 + 2 \tan t$; $\sec t = \frac{x+1}{3}$ and $\tan t = \frac{y-1}{2}$;

$\sec^2 t - \tan^2 t = 1$ yields the equation $\frac{(x+1)^2}{9} - \frac{(y-1)^2}{4} = 1$.

3. Example 1: $x = 3/\cos t$, $y = 2 \tan t$; Example 2: $x = 2 \tan t$, $y = \sqrt{5}/\cos t$; Example 3: $x = 3 + 5/\cos t$, $y = -1 + 4 \tan t$;

Example 4: $x = -2 + 3/\cos t$, $y = 5 + 7 \tan t$

5. Example 1: $x = 3/\cos t = 3 \sec t$, $y = 2 \tan t$;

$\sec t = x/3$, $\tan t = y/2$;

$\sec^2 t - \tan^2 t = 1$ yields $x^2/9 - y^2/4 = 1$, or $4x^2 - 9y^2 = 36$.

Example 2: $x - 2 \tan t$, $y = \sqrt{5}/\cos t = \sqrt{5} \sec t$;

$\tan t = x/2$, $\sec t = y/\sqrt{5}$;

$\sec^2 t - \tan^2 t = 1$ yields $y^2/5 - x^2/4 = 1$.

Example 3: $x = 3 + 5/\cos t = 3 + 5 \sec t$, $y = -1 + 4 \tan t$;

$\sec t = (x - 3)/5$, $\tan t = (y - 1)/4$;

$\sec^2 t - \tan^2 t = 1$ yields $(x - 3)^2/25 - (y + 1)^2/16 = 1$.

Example 4: $x = -2 + 3/\cos t = -2 + 3 \sec t$, $y = 5 + 7 \tan t$;

$\sec t = (x + 2)/3$, $\tan t = (y - 5)/7$;

$\sec^2 t - \tan^2 t = 1$ yields $(x + 2)^2/9 - (y - 5)^2/49 = 1$.

Quick Review 8.3

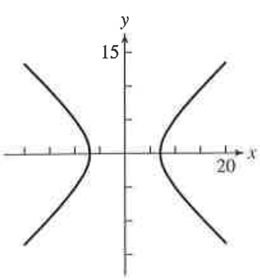
1. $\sqrt{146}$ **3.** $y = \pm\frac{4}{3}\sqrt{9 + x^2}$ **5.** No solution **7.** $x = 2, x = -2$ **9.** $a = 3, c = 5$

Exercises 8.3

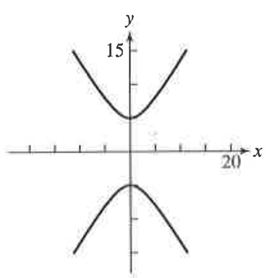
1. Vertices: $(\pm 4, 0)$; Foci: $(\pm\sqrt{23}, 0)$ **3.** Vertices: $(0, \pm 6)$; Foci: $(0, \pm 7)$

5. Vertices: $(\pm 2, 0)$; Foci: $(\pm\sqrt{7}, 0)$ **7.** (c) **9.** (a)

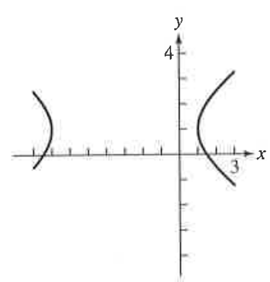
11.



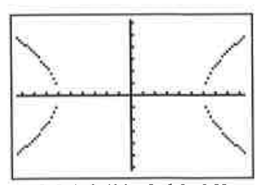
13.



15.

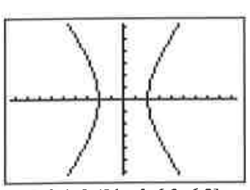


17.



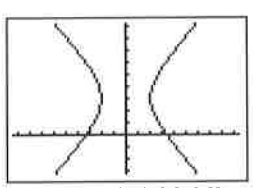
[–9.4, 9.4] by [–6.2, 6.2]

19.



[–9.4, 9.4] by [–6.2, 6.2]

21.



[–9.4, 9.4] by [–3.2, 9.2]

23. $\frac{x^2}{4} - \frac{y^2}{5} = 1$ **25.** $\frac{y^2}{16} - \frac{x^2}{209} = 1$ **27.** $\frac{x^2}{25} - \frac{y^2}{75} = 1$

29. $\frac{x^2}{144} - \frac{y^2}{25} = 1$ **31.** $\frac{(y-1)^2}{25} - \frac{(x-2)^2}{9} = 1$

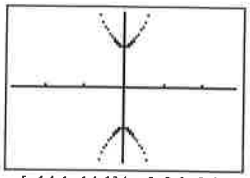
33. $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$ **35.** $\frac{(x+1)^2}{4} - \frac{(y-2)^2}{5} = 1$

37. $\frac{(y-6)^2}{25} - \frac{(x+3)^2}{75} = 1$

39. Center $(-1, 2)$; Vertices: $(11, 2)$, $(-13, 2)$; Foci: $(12, 2)$, $(-14, 2)$

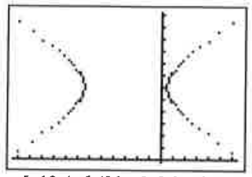
41. Center $(2, -3)$; Vertices: $(2, 5)$, $(2, -11)$; Foci: $(2, -3 \pm \sqrt{145})$

43.



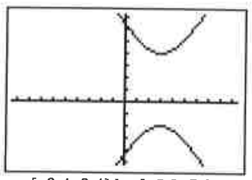
[−14.1, 14.1] by [−9.3, 9.3]

45.



[−12.4, 6.4] by [−0.2, 12.2]

47.

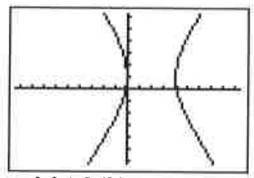


[−9.4, 9.4] by [−5.2, 7.2]

Vertices: $(3, -2)$, $(3, 4)$; Foci: $(3, 1 \pm \sqrt{13})$;

Eccentricity: $\dfrac{\sqrt{13}}{3}$

49.



[−9.4, 9.4] by [−6.2, 6.2]

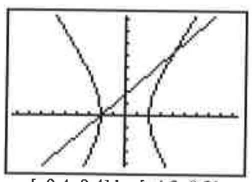
Vertices: $(0, 1)$, $(4, 1)$; Foci: $(2 \pm \sqrt{13}, 1)$;

Eccentricity: $\dfrac{\sqrt{13}}{2}$

51. $\dfrac{x^2}{4} - \dfrac{5y^2}{16} = 1$ **55.** $a = 1440$, $b = 600$, $c = 1560$, $e = 13/12$; The sun is centered at $= (1560, 0)$.

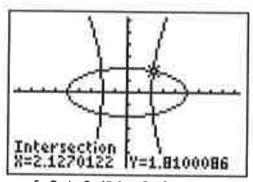
57. A bearing and distance of about 40.29° and 1371.11 miles, respectively.

59. $(-2, 0)$, $(4, 3\sqrt{3})$



[−9.4, 9.4] by [−4.2, 8.2]

61. (a)



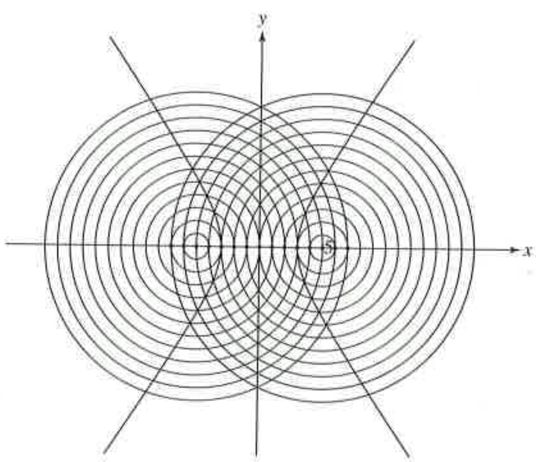
[−9.4, 9.4] by [−6.2, 6.2]

Four solutions: $(\pm 2.13, \pm 1.81)$

(b) $\left(\pm 10\sqrt{\dfrac{29}{641}}, \pm 10\sqrt{\dfrac{21}{641}}\right)$

63. True, because $c - a = ae - a$. **65.** (b) **67.** (b)

69. (a–d)



(e) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

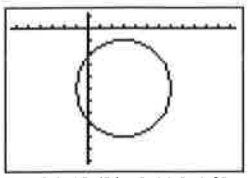
SECTION 8.4

Quick Review 8.4

1. $\cos 2\alpha = \dfrac{5}{13}$ **3.** $\cos 2\alpha = \dfrac{1}{2}$ **5.** $\alpha = \dfrac{\pi}{4}$ **7.** $\cos \alpha = \dfrac{2}{\sqrt{5}}$ **9.** $\sin \alpha = \dfrac{\sqrt{6 - \sqrt{11}}}{2\sqrt{3}}$

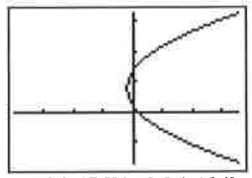
Exercises 8.4

1. $y = -5 \pm \sqrt{-x^2 + 6x + 7}$



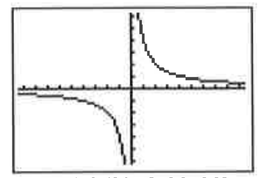
[–6.4, 12.4] by [–11.2, 1.2]

3. $y = 4 \pm \sqrt{8x + 8}$



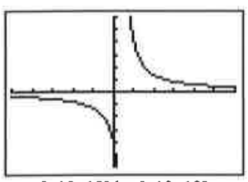
[–19.8, 17.8] by [–8.4, 16.4]

5. $y = 4/x$



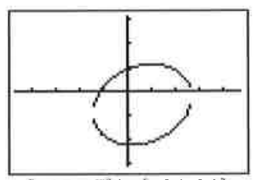
[–9.4, 9.4] by [–6.2, 6.2]

7. $y = 8/(x - 1)$



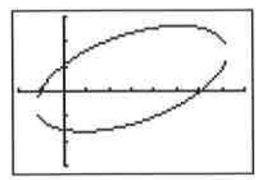
[–10, 12] by [–12, 12]

9. $y = \frac{1}{6}(x - 4 \pm \sqrt{-23x^2 + 28x + 88})$



[–4.7, 4.7] by [–3.1, 3.1]

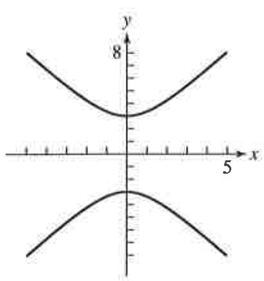
11. $y = \frac{1}{4}(x - 1 \pm \sqrt{3(-x^2 + 6x + 9)})$



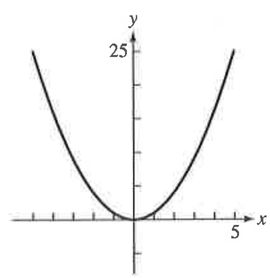
[–2, 8] by [–3, 3]

13. $x^2 = -4y$ **15.** $\frac{x^2}{9} - \frac{y^2}{16} = 1$ **17.** $(x', y') = (4, -1)$ **19.** $(x', y') = (5, -3 - \sqrt{5})$

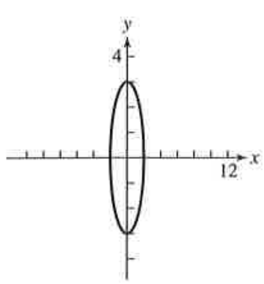
21. Hyperbola: $\frac{(y-1)^2}{9} - \frac{(x+1)^2}{4} = 1$; $\frac{(y')^2}{9} - \frac{(x')^2}{4} = 1$



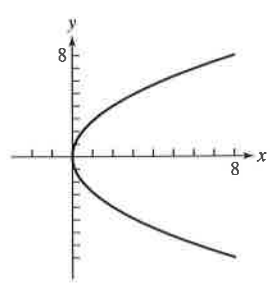
23. Parabola: $(x + 1)^2 = y - 2$; $(x')^2 = y'$



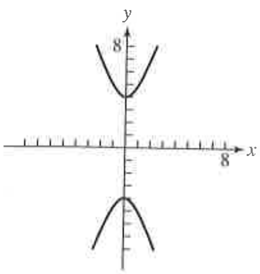
25. Elipse: $\frac{(y+2)^2}{9} + \frac{(x-1)^2}{4} = 1$; $\frac{(y')^2}{9} + \frac{(x')^2}{4} = 1$



27. Parabola: $(y - 2)^2 = 8(x - 2)$; $(y')^2 = 8x'$



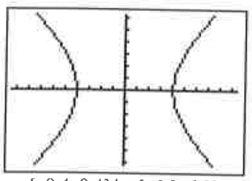
29. Hyperbola: $\frac{y^2}{4} - \frac{(x+1)^2}{2} = 1$; $\frac{(y')^2}{4} - \frac{(x')^2}{2} = 1$



31. The horizontal distance from O to P is $x = h + x' = x' + h$, and the vertical distance is $y = k + y' = y' + k$.

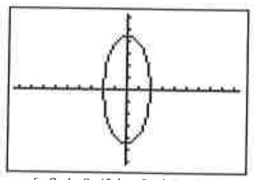
33. $\left(\frac{3\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$ **35.** $\approx (-5.94, 2.38)$

37. Hyperbola: $\frac{(x')^2}{16} - \frac{(y')^2}{16} = 1.$



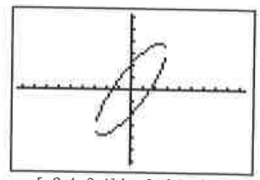
[–9.4, 9.4] by [–6.2, 6.2]

39. Ellipse: $\frac{(y')^2}{20} + \frac{(x')^2}{4} = 1$



[–9.4, 9.4] by [–6.2, 6.2]

41. Ellipse: $y = \frac{10x \pm 2\sqrt{90 - 11x^2}}{9}$



[–9.4, 9.4] by [–6.2, 6.2]

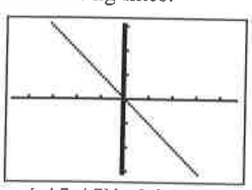
$\alpha \approx 0.954 \approx 54.65°$

43. $-24 < 0$; ellipse **45.** $-71 < 0$; ellipse **47.** $-48 < 0$; ellipse **49.** $12 > 0$; hyperbola **51.** $-12 < 0$; ellipse

53. In the "old" coordinate system, the center is (0, 0), the vertices are $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, and $\left(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$, and the foci are (3, 3) and $(-3, -3)$. **57.** True, because the xy term is missing. **59.** (b) **61.** (a)

63. **(a)** $y = \pm x$ **(b)** $y = 2x + 3/2$, $y = (21 - x)/2$

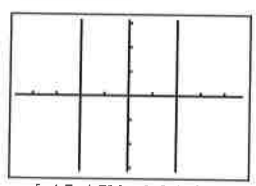
69. Intersecting lines:



[–4.7, 4.7] by [–3.1, 3.1]

A plane containing the axis of a cone intersects the cone.

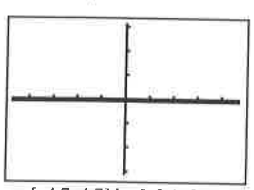
Parallel lines:



[–4.7, 4.7] by [–3.1, 3.1]

A degenerate cone is created by a generator that is parallel to the axis, producing a cylinder. A plane parallel to a generator of the cylinder intersects the cylinder and its interior.

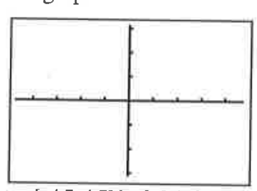
One line:



[–4.7, 4.7] by [–3.1, 3.1]

A plane containing a generator of a cone intersects the cone.

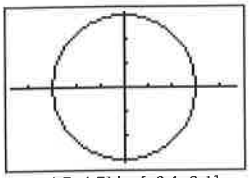
No graph:



[–4.7, 4.7] by [–3.1, 3.1]

A plane parallel to a generator of a cylinder fails to intersect the cylinder. Also, a degenerate cone is created by a generator that is perpendicular to the axis, producing a plane. A second plane perpendicular to the axis of this degenerate cone fails to intersect it.

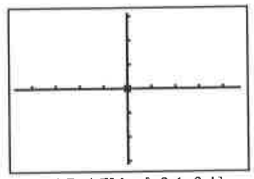
Circle:



[−4.7, 4.7] by [−3.1, 3.1]

A plane perpendicular to the axis of a cone intersects the cone but not its vertex.

Point:



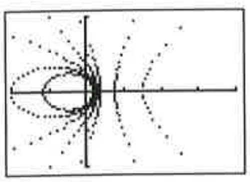
[−4.7, 4.7] by [−3.1, 3.1]

A plane perpendicular to the axis of a cone intersects the vertex of the cone.

SECTION 8.5

Exploration 1

1.



[−12, 24] by [−12, 12]

$e = 0.7$, $e = 0.8$, an ellipse; $e = 1$, a parabola; $e = 1.5$, $e = 3$, a hyperbola.
The graphs have common focus, (0, 0), and a common directrix, the line $x = 3$.
As e increases, the graphs move away from the focus and toward the directrix.

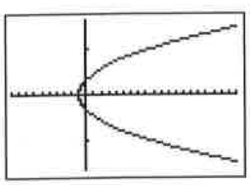
One possible answer: The graphs are similar in that they are all conic sections with a common vertex at (1.24, 0). They are different, however, in that values of $e < 1$ produce a closed figure (an ellipse), $e = 1$ produces a parabola and $e > 1$ produce hyperbolas.

Quick Review 8.5

1. $r = -3$ **3.** $\theta = \frac{7\pi}{6}, \theta = -\frac{5\pi}{6}$ **5.** The focus is (0, 4) and the directrix is $y = -4$.
7. Foci: $(\pm\sqrt{5}, 0)$; Vertices: $(\pm 3, 0)$ **9.** Foci: $(\pm 5, 0)$; Vertices: $(\pm 4, 0)$

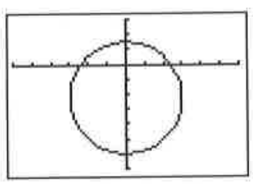
Exercises 8.5

1. $r = \frac{2}{1 - \cos\theta}$; Parabola



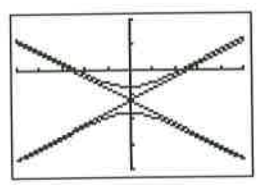
[−10, 20] by [−10, 10]

3. $r = \frac{12}{5 + 3\sin\theta}$; Ellipse



[−6, 6] by [−7, 3]

5. $r = \frac{7}{3 - 7\sin\theta}$; Hyperbola



[−5, 5] by [−4, 2]

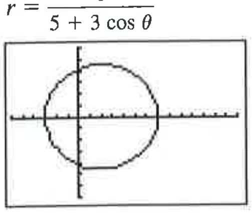
7. $e = 1$, Parabola; Directrix: $x = 2$ **9.** $e = 1$, Parabola; Directrix: $y = -\frac{5}{2} = -2.5$ **11.** $e = \frac{5}{6}$, Ellipse; Directrix: $y = 4$

13. $e = \frac{2}{5} = 0.4$, Ellipse; Directrix: $x = 3$ **15.** (b); [−15, 5] by [−10, 10] **17.** (f); [−5, 5] by [−3, 3]

19. (c); [−10, 10] by [−5, 10] **21.** $r = \frac{12}{5 + 3\cos\theta}$ **23.** $r = \frac{3}{2 + \sin\theta}$ **25.** $r = \frac{15}{2 + 3\cos\theta}$ **27.** $r = \frac{12}{2 + 3\sin\theta}$

29. $r = \frac{6}{5 + 3\cos\theta}$

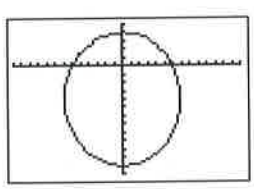
31.



[−6, 14] by [−7, 6]

$e = 0.4$, $a = 5$, $b = \sqrt{21}$, $c = 2$

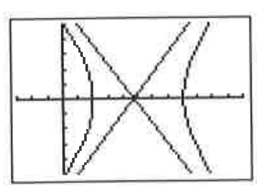
33.



[−13, 14] by [−13, 5]

$e = \frac{1}{2}$, $a = 8$, $b = 4\sqrt{3}$, $c = 4$

35.



[−3, 12] by [−5, 5]

$e = \frac{5}{3}$, $a = 3$, $b = 4$, $c = 5$

37. $\frac{9(y - 4/3)^2}{64} + \frac{3x^2}{16} = 1$ **39.** $y^2 = 4(x + 1)$ **41.** Perihelion distance $\approx$ 0.54 AU; Aphelion distance $\approx$ 35.64 AU

43. (a) $v \approx 1551$ m/sec $= 1.551$ km/sec **(b)** about 2 hr 14 min **45.** True. For a circle, $e = 0$, so the equation is $r = 0$, which graphs as a point. **47.** (d) **49.** (b)

51. (c)

Planet	Perihelion Distance (AU)	Aphelion Distance (AU)
Mercury	0.307	0.467
Venus	0.718	0.728
Earth	0.983	1.017
Mars	1.382	1.665
Jupiter	4.953	5.452
Saturn	9.020	10.090

(d) The difference is greatest for Saturn.

SECTION 8.6

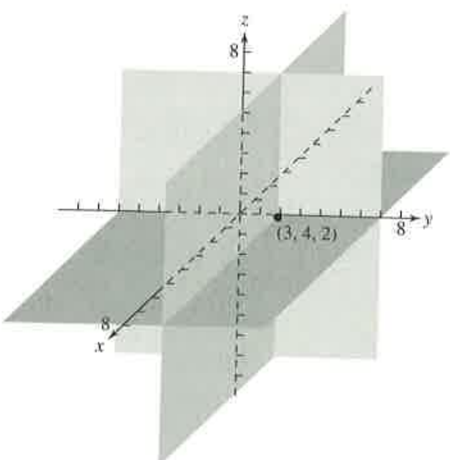
Quick Review 8.6

1. $\sqrt{(x - 2)^2 + (y + 3)^2}$ **3.** P lies on the circle of radius 5 centered at $(2, -3)$. **5.** $\left\langle \frac{-4}{\sqrt{41}}, \frac{5}{\sqrt{41}} \right\rangle$

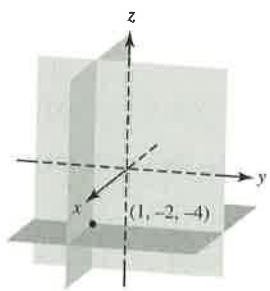
7. Circle of radius 5 centered at $(-1, 5)$ **9.** Center: $(-1, 3)$, Radius: 2

Exercises 8.6

1.



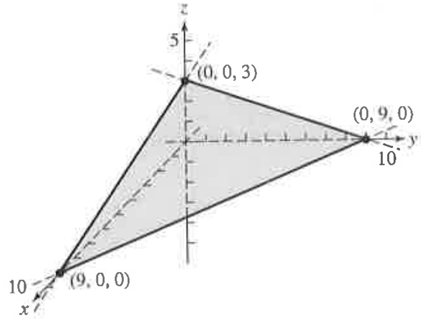
3.



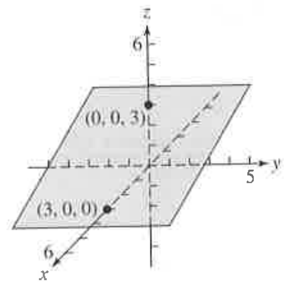
5. $\sqrt{53}$ **7.** $\sqrt{(a - 1)^2 + (b + 3)^2 + (c - 2)^2}$ **9.** $\left(1, -1, \frac{11}{2}\right)$ **11.** $(x - 1, y + 4, z + 3)$

13. $(x - 5)^2 + (y + 1)^2 + (z + 2)^2 = 64$ **15.** $(x - 1)^2 + (y + 3)^2 + (z - 2)^2 = a$

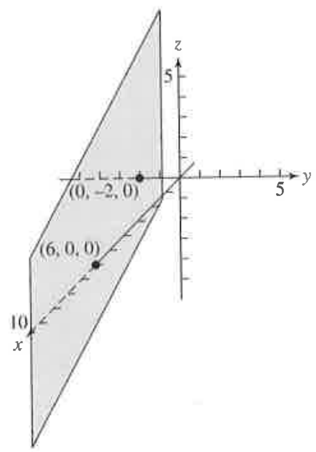
17.



19.



21.



23. $\langle -2, 4, -8\rangle$ **25.** -84 **27.** -20 **29.** $\left\langle \frac{4}{13}, -\frac{3}{13}, \frac{12}{13}\right\rangle$

31. $\langle -3, 4, -5\rangle$ **33.** $\mathbf{v} = -195.01\mathbf{i} - 7.07\mathbf{j} + 68.04\mathbf{k}$

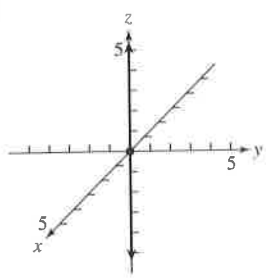
35. $\mathbf{r} = (2, -1, 5) + t(3, 2, -7)$; $x = 2 + 3t, y = -1 + 2t, z = 5 - 7t$

37. $\mathbf{r} = (6, -9, 0) + t(1, 0, -4)$; $x = 6 + t, y = -9, z = -4t$ **39.** $\sqrt{30}$

41. $\mathbf{r} = \langle -1, 2, 4\rangle + t\langle 1, 4, -7\rangle$ **43.** $x = -1 + 3t, y = 2 - 6t, z = 4 - 3t$

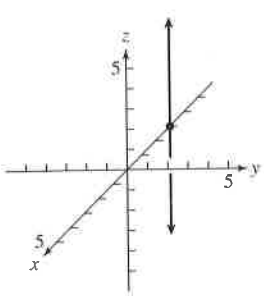
45. $x = \frac{1}{2}t, y = 6 - 7t, z = -3 + \frac{11}{2}t$ **47.** scalene

49. (a)



(b) the z-axis; a line through the origin in the direction $\mathbf{k}$

51. (a)



(b) the intersection of the xz plane ($y = 0$) and the plane $x = -3$; a line parallel to the z-axis through $(-3, 0, 0)$

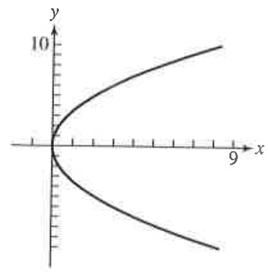
53. $\mathbf{r} = \langle x_1 + (x_2 - x_1)t, y_1 + (y_2 - y_1)t, z_1 + (z_2 - z_1)t\rangle$ **55.** By the Pythagorean theorem, $d(P, Q) = \sqrt{((d(P, R))^2 + (d(R, Q))^2}$ $= \sqrt{(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)})^2 + (|z_1 - z_2|)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

57. True. This is the equation of a vertical elliptical cylinder. **59.** (b) **61.** (c) **65.** $\langle -1, -5, -3\rangle$

67. $\mathbf{i} \times \mathbf{j} = \langle 1, 0, 0\rangle \times \langle 0, 1, 0\rangle = \langle 0 - 0, 0 - 0, 1 - 0\rangle = \langle 0, 0, 1\rangle = \mathbf{k}$

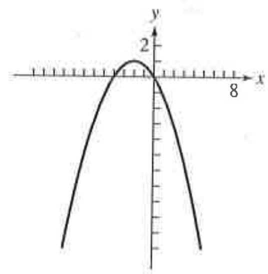
CHAPTER 8 REVIEW EXERCISES

1.



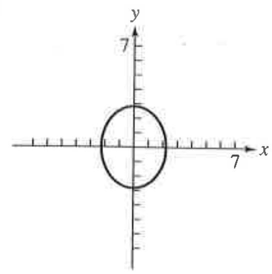
Vertex: (0, 0); Focus: (3, 0);
Directrix: $x = -3$; Focal width: 12

3.



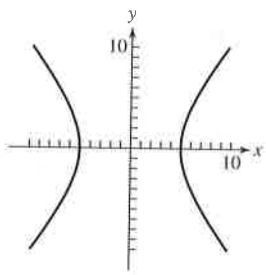
Vertex: (−2, 1); Focus: (−2, 0);
Directrix: $y = 2$; Focal width: 4

5.



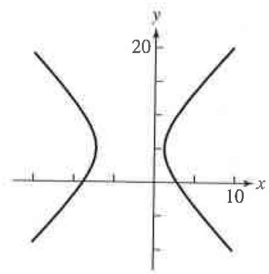
Ellipse; Center: (0, 0);
Vertices: $(0, \pm 2\sqrt{2})$; Foci: $(0, \pm\sqrt{3})$

7.



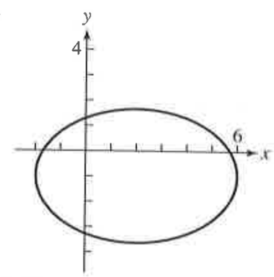
Hyperbola; Center: (0, 0);
Vertices: (±5, 0); Foci: $(\pm\sqrt{61}, 0)$

9.



Hyperbola; Center: (−3, 5); Vertices:
$(-3 \pm 3\sqrt{2}, 5)$; Foci: $(-3 \pm \sqrt{46}, 5)$

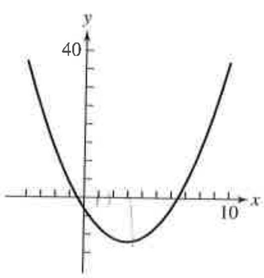
11.



Ellipse; Center: (2, −1);
Vertices: (6, −1), (−2, −1);
Foci: (5, −1), (−1, −1)

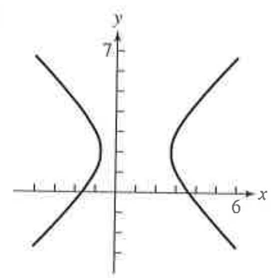
13. (b) **15.** (h) **17.** (f) **19.** (c)

21.



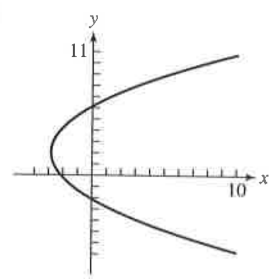
Parabola; $(x - 3)^2 = y + 1$

23.



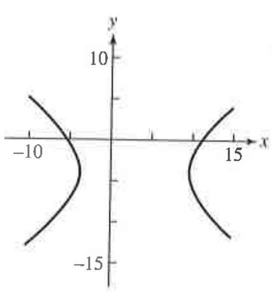
Hyperbola; $\frac{(x-1)^2}{3} - \frac{(y-2)^2}{3} = 1$

25.



Parabola; $(y - 2)^2 = 6\left(x + \frac{17}{6}\right)$

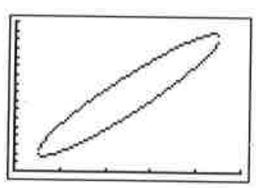
27.



Hyperbola; $\frac{(y+4)^2}{30} - \frac{(x-3)^2}{45} = 1$

29. See proof on page 633.

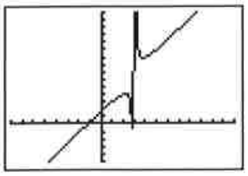
31.



[0, 25] by [0, 17]

Ellipse; $y = \frac{1}{12}[8x + 5 \pm \sqrt{-8x^2 + 200x - 455}]$

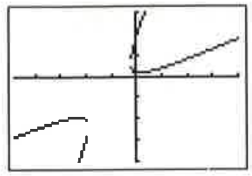
33.



[−8, 12] by [−5, 15]

Hyperbola; $y = \frac{3x^2 - 5x - 10}{2x - 6}$

35.



[−24, 20] by [−20, 15]

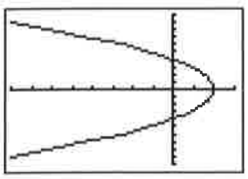
Hyperbola; $y = \frac{1}{4}[7x + 20 \pm \sqrt{25x^2 + 272x + 280}]$

37. $y^2 = 8x$ **39.** $(x + 3)^2 = 12(y - 3)$ **41.** $\frac{x^2}{169} + \frac{y^2}{25} = 1$ **43.** $\frac{x^2}{9} + \frac{(y - 2)^2}{5} = 1$ **45.** $\frac{y^2}{25} - \frac{x^2}{11} = 1$

47. $\frac{(x - 2)^2}{9} - \frac{(y - 1)^2}{16} = 1$ **49.** $\frac{x^2}{25} + \frac{y^2}{4} = 1$ **51.** $(x - 2)^2 + (y - 4)^2 = 1$

53. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

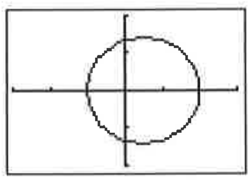
55.



[−8, 3] by [−10, 10]

Parabola; $y^2 = -8(x - 2)$

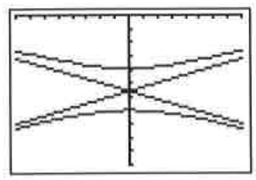
57.



[−3, 3] by [−2, 2]

Ellipse; $\frac{4(x - 1/2)}{9} + \frac{y^2}{2} = 1$

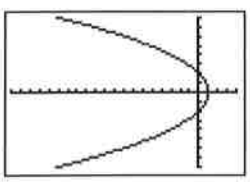
59.



[−8, 8] by [−11, 0]

Hyperbola; $\frac{81(y + 49/9)^2}{196} - \frac{9x^2}{245} = 1$

61.



[−20, 4] by [−8, 8]

Parabola; $y^2 = -4(x - 1)$

63. $\sqrt{69}$ **65.** $\langle 0, -3, -2 \rangle$ **67.** -13 **69.** $\langle 3/5, -4/5, 0 \rangle$

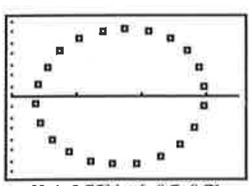
71. $(x + 1)^2 + y^2 + (z - 3)^2 = 16$ **73.** $\mathbf{r} = \langle -1, 0, 3 \rangle + t\langle -3, 1 -2 \rangle$ **75.** (0, 4.5)

79. At apogee, $v \approx 2633$ m/sec. At perigee, $v \approx 9800$ m/sec

Chapter 8 Project

Answers are based on the sample data provided.

1.



[0.4, 0.75] by [−0.7, 0.7]

3. With respect to the graph of the ellipse, the point (h, k) represents the center of the ellipse. The value a is the length of the semimajor axis and b is the length of the semiminor axis.

5. 0.978