Name_____ Date_____ Block_____

Exponential – Logarithmic – Logistic Modeling Problems

1. A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling so that its temperature at time t is given by $T(t) = 65 + 145\, e^{-0.05t}$, where t is measured in minutes and T is measured in °F.

   a) What is the initial temperature of the soup?

   b) What is the temperature after 10 minutes?

   c) After how long will the temperature be 100°F?

2. Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death, the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling for humans is approximately $k = 0.1947$, assuming time is measured in hours. Suppose the temperature of the surroundings is 60°F.

   a) Find a function T(t) that models the temperature t hours after death.

   b) If the temperature of the body is now 72°F, how long ago was the time of death?

3. The logistic growth model $P(t) = \frac{0.9}{1 + 3.5\, e^{-0.339t}}$ relates the proportion of new personal computers sold at Best Buy that have Intel's latest coprocessor t months after it has been introduced.

   a) What proportion of new personal computers sold at Best Buy will have Intel's latest coprocessor when it is first introduced (when $t = 0$)?

   b) Determine the maximum proportion of new personal computers sold at Best Buy that will have Intel's latest coprocessor.

   c) When will 75% of new personal computers sold at Best Buy have Intel's latest coprocessor?

4. The logistic model $P(t) = \frac{1000}{1 + 32.33\, e^{-0.439t}}$ relates to the population of a bacteria after t hours.

   a) What is the carrying capacity of the environment?

   b) What was the initial amount of bacteria in the population?

   c) When will the amount of bacteria be 800?

5. The half-life of a radioactive material can be found by using the formula: $N = N_0 e^{-kt}$ where k is a constant unique to the material and t is measured in years. Find the half-life of Plutonium-238 given that its constant k is 0.008022.

6. Find the constant k for Plutonium-239 given that its half-life is 24,390 years.

7. As long as a plant or animal is alive, carbon 14 is maintained in a constant amount in its tissues. Once dead, however, the plant or animal ceases taking in carbon, and carbon 14 diminishes radioactive decay. Estimate the age of a skull uncovered in an archaeological site if 10% of the original amount of carbon 14 is still present. if $k = .00165$

8. How long must a principle amount of \$17,245 stay in an account earning 4.25% interest compounded continuously before \$25,000 is attained?

9. Lucas has \$8,000 to save in a savings account. His goal is to have \$12,000 after 2 years of continuous compounding. At what interest rate must the bank offer for this to be possible?

10. Sal is going to invest \$3500 for 3 years. Which of the following choices will earn him the highest profit?

    a) $8\frac{7}{8}$ % compounded annually,

    b) 8 ½ % compounded quarterly, or

    c) 8 ¼ % compounded continuously?

11. The population of the world was 5.2 billion in 1990. The exponential growth rate was 1.6% per year.

    a) Find the exponential growth function

    b) Estimate the population of the world in 2000

    c) When will the world population be 8.0 billion?

12. A certain satellite has a power supply whose output in watts is given by the equation: $P = 40e^{(-t/900)}$, where t is the number of days the battery has operated.

    a) If it is operated continuously after the satellite is placed into orbit, how many watts is the battery putting out after one year?

    b) If it takes at least 10 watts to operate the satellite, how many days can the satellite be used?

13. The hydrogen ion concentration of fresh egg whites was measured to be $[H^+] = 1.2 \times 10^{-8}$ M. Find the pH and classify the substance as acidic or basic.

14. The pH of limejuice is 2.1. Find the hydrogen ion concentration.

15. The noise level at a rock concert was measured at 135 dB. Find the intensity in watts per square meter.

16. Find the loudness of a garbage disposal unit that operates at an intensity of 10^{-4} watts per square meter. Express your answer in decibels.

$$A = P(1+\frac{r}{n})^{nt} \qquad A = Pe^{rt} \qquad A = A_o e^{kt}$$

$$T = T_s + D_o e^{-kt} \quad \text{or } T(t) = T_m + (T_o - T_m)e^{-kt} \qquad pH = -\log\left[H^+\right] \qquad dB = 10\log\left(\frac{I}{1\times10^{-12}}\right)$$