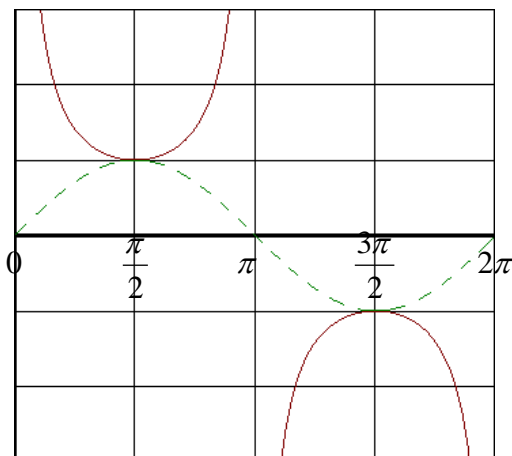


## 6.5 Graphs of Tan(x), Cot(x), Csc(x), and Sec(x)

Now we will look at other types of graphs:  $\tan x$ ,  $\cot x$ ,  $\csc x$ ,  $\sec x$ . We will start with the cosecant and secant since it follows from the previous section.

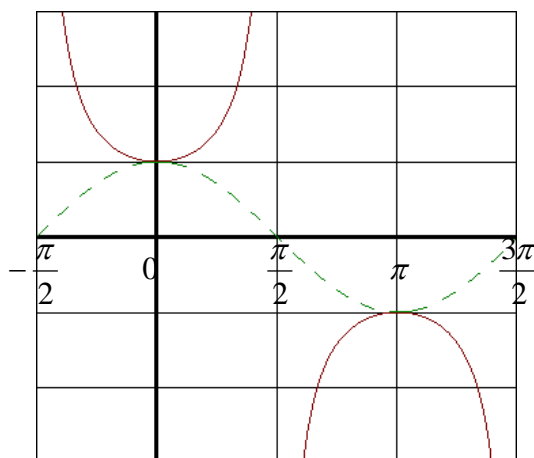
Graph of  $y = \csc x$



In order to draw this graph we will first start with the graph of  $y = \sin x$  (dotted line). Wherever the graph  $y = \sin x$  crosses the x-axis is where there is a vertical asymptote.

The period of  $y = \csc x$  is  $2\pi$ . The amplitude is 1, since the graph touches  $y = \sin x$  at its amplitude.

Graph of  $y = \sec x$



In order to draw this graph we will first start with the graph of  $y = \cos x$  (dotted line). Wherever the graph  $y = \cos x$  crosses the x-axis is where there is a vertical asymptote.

The period of  $y = \sec x$  is  $2\pi$ . The amplitude is 1, since the graph touches  $y = \cos x$  at its amplitude.

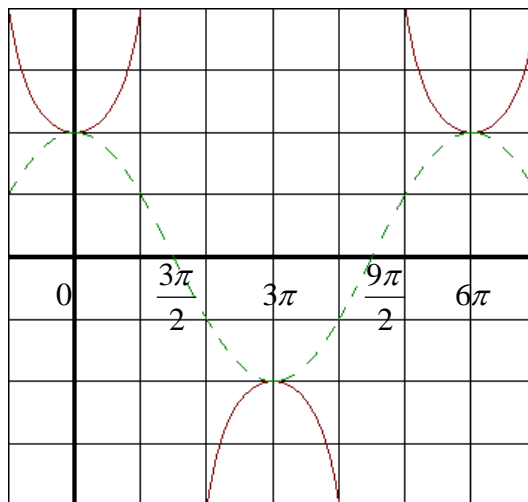
EXAMPLE: Graph  $y = 2\sec\left(\frac{x}{3}\right)$  over one period.

We will first pretend this is a cosine function. The period of this is:  $\text{Period} = \frac{2\pi}{B}$ . In this problem, the period is  $\frac{2\pi}{1/3} = 6\pi$ . To find the phase shift, we can rewrite our problem as  $y = 2\sec\left(\frac{1}{3}x + 0\right)$ . Then when we apply the formula for the phase shift, we get zero. This means the graph does not move left or right. Next we can

find the increment, which is the period divided by 4: In this case our increment is  $\frac{6\pi}{4} = \frac{3\pi}{2}$ . We start from zero and keep adding the increment as we did in the previous section.

$$0 + \frac{3\pi}{2} = \frac{3\pi}{2} \qquad \frac{3\pi}{2} + \frac{3\pi}{2} = \frac{6\pi}{2} \qquad \frac{6\pi}{2} + \frac{3\pi}{2} = \frac{9\pi}{2} \qquad \frac{9\pi}{2} + \frac{3\pi}{2} = \frac{12\pi}{2}$$

So now we have all of our key points, so we can put them all on the graph. Next, we will draw a dotted line to represent the cosine function. Where ever the cosine graph crosses the x-axis is where there will be a vertical asymptote. So you can draw a vertical dotted line through each place it hits the x-axis. Finally we will draw in the curves for the secant function. The final graph will look like this:



EXAMPLE: Graph  $y = -3\csc\left(x - \frac{\pi}{4}\right)$  over one period.

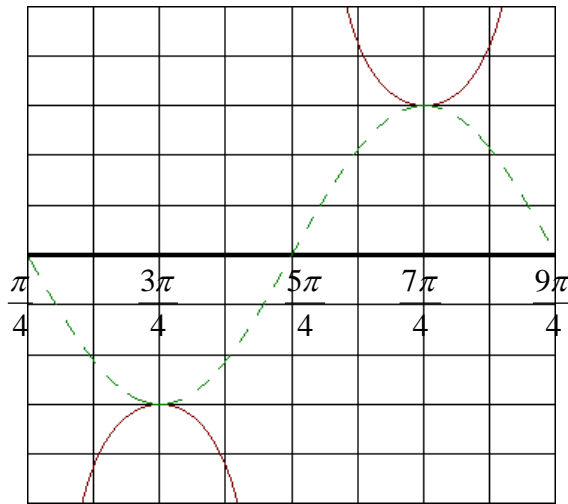
We will first pretend this is a sine function. The period of this is:  $\text{Period} = \frac{2\pi}{B}$ . In this problem, the period is

$\frac{2\pi}{1} = 2\pi$ . To find the phase shift, we take the opposite sign of c and divide it by b:  $\frac{\pi/4}{1} = \frac{\pi}{4}$ . This will be our first key point. Next we can find the increment, which is the period divided by 4: In this case our increment is  $\frac{2\pi}{4}$ . I will leave it in the unreduced form to make it easier to add since you will have common denominators

already. We start from  $\frac{\pi}{4}$  and keep adding the increment as we did in the previous section.

$$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4} \qquad \frac{3\pi}{4} + \frac{2\pi}{4} = \frac{5\pi}{4} \qquad \frac{5\pi}{4} + \frac{2\pi}{4} = \frac{7\pi}{4} \qquad \frac{7\pi}{4} + \frac{2\pi}{4} = \frac{9\pi}{4}$$

So now we have all of our key points, so we can put them all on the graph. Next, we will draw a dotted line to represent the sine function. Where ever the sine graph crosses the x-axis is where there will be a vertical asymptote. So you can draw a vertical dotted line through each place it hits the x-axis. Finally we will draw in the curves for the secant function. The final graph will look like this:

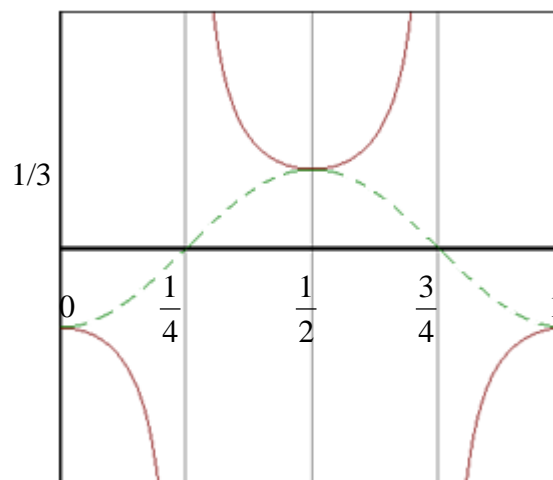


EXAMPLE: Graph  $y = -\frac{1}{3}\sec(2\pi x)$  over one period.

We will first pretend this is a cosine function. The period of this is:  $\text{Period} = \frac{2\pi}{B}$ . In this problem, the period is  $\frac{2\pi}{2\pi} = 1$ . To find the phase shift, we can rewrite our problem as  $y = -\frac{1}{3}\sec(2\pi x + 0)$ . Then when we apply the formula for the phase shift, we get zero. This means the graph does not move left or right. Next we can find the increment, which is the period divided by 4: In this case our increment is  $\frac{1}{4}$ . We start from zero and keep adding the increment as we did in the previous section.

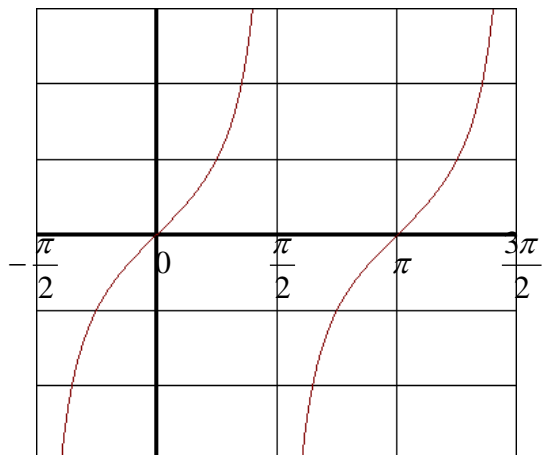
$$0 + \frac{1}{4} = \frac{1}{4} \quad \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \quad \frac{3}{4} + \frac{1}{4} = 1$$

So now we have all of our key points, so we can put them all on the graph. Next, we will draw a dotted line to represent the cosine function. Where ever the cosine graph crosses the x-axis is where there will be a vertical asymptote. So you can draw a vertical dotted line through each place it hits the x-axis. Finally we will draw in the curves for the secant function. The final graph will look like this:



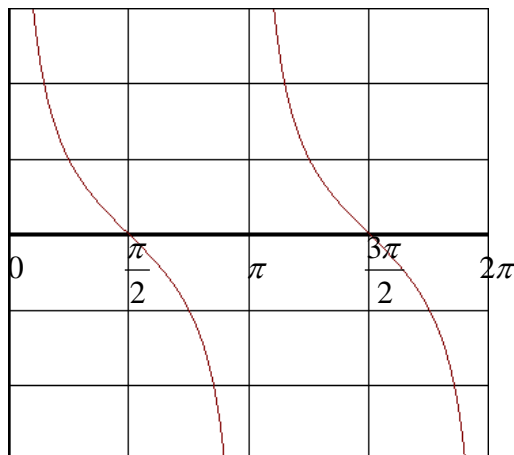
Now we will look at the tangent and cotangent graphs.

Graph of  $y = \tan x$



The period is  $\pi$ . There are vertical asymptotes at  $x = \frac{n\pi}{2}$  where  $n$  is any integer.

Graph of  $y = \cot x$



The period is  $\pi$ . There are vertical asymptotes at  $x = n\pi$  where  $n$  is any integer. Notice the graph goes the opposite direction as the tangent graph.

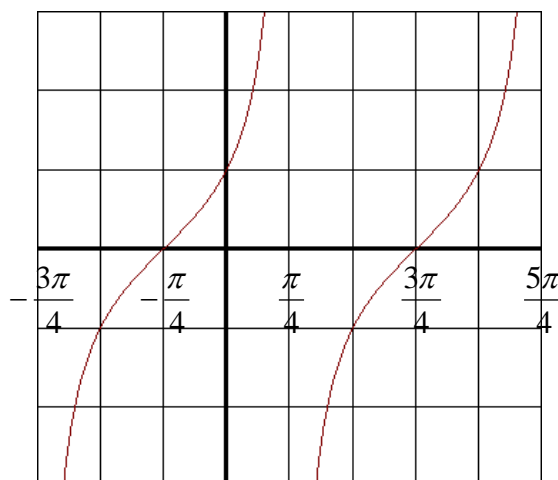
EXAMPLE: Graph  $y = \tan\left(x + \frac{\pi}{4}\right)$  over 2 periods.

Because we have a positive number after the  $x$ , this means the graph will be shifted to the left an amount of  $\frac{\pi}{4}$ .

Therefore all the original key points will move to the left. So what we need to do is subtract  $\frac{\pi}{4}$  from all of our original key points. This will give us a new set of points we can graph:

$$-\frac{\pi}{2} - \frac{\pi}{4} = -\frac{3\pi}{4} \quad 0 - \frac{\pi}{4} = -\frac{\pi}{4} \quad \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \quad \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4}$$

Notice that the asymptotes will be the first, third, and fifth key points, just like in the original graph.



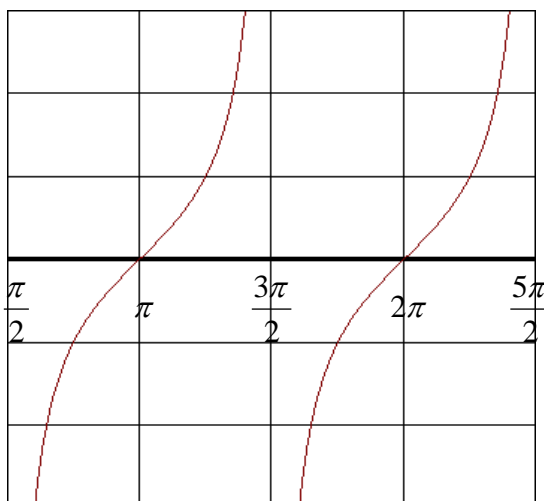
EXAMPLE: Graph  $y = -\cot\left(x - \frac{\pi}{2}\right)$  over two periods.

Because we have a negative number after the  $x$ , this means the graph will be shifted to the right an amount of  $\frac{\pi}{2}$ . Therefore all the original key points will move to the right. So what we need to do is add  $\frac{\pi}{2}$  from all of our original key points. This will give us a new set of points we can graph:

$$0 + \frac{\pi}{2} = \frac{\pi}{2} \quad \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad \pi + \frac{\pi}{2} = \frac{3\pi}{2} \quad \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi \quad 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$

Notice that the asymptotes will be the first, third, and fifth key points, just like in the original graph.

This time, there is a negative sign in front of the cotangent. This means we need to flip our graph. So as you can see in the graph below, the graph actually resembles a tangent graph because of the flip.



EXAMPLE:  $y = -2 \tan\left(\frac{\pi}{3}x\right)$

If you have another number in front of the x, this means the period will change. The formula for finding the period is  $\frac{\pi}{B}$ , where B is the number in front of the x. So for this problem,  $\text{Period} = \frac{\pi}{\pi/3} = \frac{\pi}{1} \cdot \frac{3}{\pi} = 3$ . Notice

that we took the original period, which was  $\pi$ , and we multiplied this by  $\frac{3}{\pi}$ . This means we need to do this to

all our original key points for the tangent graph. Take each original key point and multiply each of them by  $\frac{3}{\pi}$ :

$$-\frac{\pi}{2} \cdot \frac{3}{\pi} = -\frac{3}{2} \quad 0 \cdot \frac{3}{\pi} = 0 \quad \frac{\pi}{2} \cdot \frac{3}{\pi} = \frac{3}{2} \quad \pi \cdot \frac{3}{\pi} = 3 \quad \frac{3\pi}{2} \cdot \frac{3}{\pi} = \frac{9}{2}$$

Now we can label these key points on the graph and draw in our tangent. Notice the graph is flipped because of the negative sign in front of the tangent. Now the number 2 in front of the graph will stretch it, so it will be narrower than the others. The only way to highlight this is by plotting points. In MyMathLab this is not necessary because you are choosing graphs by multiple choice. On my test I will not give you a number in front of tangent or cotangent graphs. I am only looking for a basic sketch that does not require you to plot points. Here is the final graph:

