MATRIX ALGEBRA

Matrix a rectangular array of numbers written within brackets; usually named by capital letters.

$$\underset{2\times3}{A} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix}$$

$A_{2\times3}$ – columns, rows

Matrix elements every item listed in a matrix

$$\underset{3\times2}{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$$

Square Matrix any $m \times n$ matrix with the same number of rows & columns

Identity Matrix I is a square matrix with the diagonal entries are 1s and all the other entries are Øs.

$$\underset{3\times3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scanned by CamScanner

Sum and Difference of Two Matrices

Let A and B be matrices of order $m \times n$.

1. $A + B = [a_{ij} + b_{ij}]$
2. $A - B = [a_{ij} - b_{ij}]$

* Must be the same size *

Given: $A = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & 1 \\ 3 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 & 0 \\ 4 & 6 & 9 \\ -5 & 1 & -2 \end{bmatrix}$

3x3 3x3

EX #1: Add and subtract.

A. Find $A + B$

$$A+B = \begin{bmatrix} 2+5 & 4-2 & -3+0 \\ -1+4 & 0+6 & 1+9 \\ 3-5 & 2+1 & -2-2 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -3 \\ 3 & 6 & 10 \\ -2 & 3 & -4 \end{bmatrix}$$

B. Find $A - B$

$$A-B = \begin{bmatrix} 2-5 & 4+2 & -3-0 \\ -1-4 & 0-6 & 1-9 \\ 3+5 & 2-1 & -2+2 \end{bmatrix} = \begin{bmatrix} -3 & 6 & -3 \\ -5 & -6 & -8 \\ 8 & 1 & 0 \end{bmatrix}$$

Scanned by CamScanner

Operations Using Matrices

Let A, B, and C be matrices of order $m \times n$.

1. $A+B=B+A$ Commutative Property
2. $(A+B)+C=A+(B+C)$ Associative Property
3. $A+0=A$ Zero Matrix Property
4. $k(A+B)=k(A)+k(B)$ Scalar Multiplication

Given: $A = \begin{bmatrix} 5 & 1 & 3 \\ 6 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 & 4 \\ -3 & 1 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 12 & 0 \\ -4 & 8 \end{bmatrix}$

2X3 2X3 2X2

EX #2: Find the following:

A. $2A - 3B$

$$2\begin{bmatrix} 5 & 1 & 3 \\ 6 & 0 & -2 \end{bmatrix} - 3\begin{bmatrix} 0 & 2 & 4 \\ -3 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 10-0 & 2-6 & 6-12 \\ 12+9 & 0-3 & -4-24 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -4 & -6 \\ 21 & -3 & -28 \end{bmatrix}$$

B. $\frac{1}{4}C = \frac{1}{4}\begin{bmatrix} 12 & 0 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

Scanned by CamScanner

The Product of Two Matrices

PRODUCT OF TWO MATRICES

$A \times B$

$m \times n = n \times p$

The columns of A MUST equal the rows of B to find the product of 2 matrices

$AB_{m \times p}$

Row of A multiplied to Column of B

EX #3: Multiply RC given: $R = \begin{bmatrix} 3 & -4 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$

$1 \times 3 = \checkmark \; 3 \times 1$

$$RC_{1 \times 1} = \begin{bmatrix} 6 + (-24) + (-20) \\ 3(2) + (-4)(6) + 5(-4) \\ -38 \end{bmatrix}$$

$$RC = \begin{bmatrix} -38 \end{bmatrix}$$

Scanned by CamScanner

Multiplying Two Matrices

Given: $A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$

$2\times3 \quad \checkmark \quad 3\times2$

EX #4: Multiply *AB*

$$\underset{2\times2}{AB} = \begin{bmatrix} \overset{6+1+6}{3(2)+1(1)+2(3)} & \overset{0+1+4}{3(0)+1(1)+2(2)} \\ \overset{-2+1+0}{-1(2)+1(1)+0(3)} & \overset{0+1+0}{-1(0)+1(1)+0(2)} \end{bmatrix}$$

$$\underset{2\times2}{AB} = \begin{bmatrix} 13 & 5 \\ -1 & 1 \end{bmatrix}$$

EX #5: Multiply *BA*

$B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 3 & 2 \end{bmatrix} \quad A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$

$3\times2 \quad \checkmark \quad 2\times3$

$$\underset{3\times3}{BA} = \begin{bmatrix} \overset{6}{2(3)+0(-1)} & \overset{2}{2(1)+0(1)} & \overset{4}{2(2)+0(0)} \\ \overset{2}{1(3)+1(-1)} & \overset{2}{1(1)+1(1)} & \overset{2}{1(2)+1(0)} \\ \overset{7}{3(3)+2(-1)} & \overset{5}{3(1)+2(1)} & \overset{6}{3(2)+2(0)} \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 & 4 \\ 2 & 2 & 2 \\ 7 & 5 & 6 \end{bmatrix}$$

Scanned by CamScanner

Determinant of a Square Matrix

INVERSE OF A 2 × 2 MATRIX $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$

determinant $= ad - bc$

if det. $\neq 0$, $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

if det $= 0$ no inverse

EX #6: Find the inverse of matrix A. $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

det. $3(-1) - (2)(1) = -5$

$$A^{-1} = \frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 2/5 \\ 1/5 & -3/5 \end{bmatrix}$$

Scanned by CamScanner