Pre-Calculus Released NCFE (Spring 2013)

Name: Key

Arithmetic Sequence and Series

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence and Series

$$a_n = a_1 \cdot r^{(n-1)}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}, \text{ where } r \neq 1$$

$$S = \frac{a_1}{1-r}, \text{ where } |r| < 1$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Conic Sections

Parabola

Focal Length

$$|a| = \frac{1}{4c}$$

Ellipse

Pythagorean Relationship

$$c^2 = a^2 - b^2$$

Hyperbola with Center (h,k)

Pythagorean Relationship

$$c^2 = a^2 + b^2$$

Foci

$(h \pm c, k)$ or $(h, k \pm c)$

1 What are the ***approximate*** rectangular coordinates for the point with polar coordinates (5, 30°)?

(X, Y)

R θ

A (2.5, 2.89)

B (2.5, 4.33)

C (2.89, 4.33)

(D) (4.33, 2.5)

D

$X = R\cos\theta$ $Y = R\sin\theta$

$X = 5\cos 30°$ $Y = 5\sin 30°$

$X = 4.33$ $Y = 2.5$

Know formulas!
Degree mode!

2 A sequence is shown below.

6, 12, 20, 30, 42, 56, . . .

Which is the recursive formula for this sequence?

~~A~~ $t_n = n + 2(t_{n-1} + 1)$

~~B~~ $t_n = (t_{n-1} + 1)(n - 2)$

C $t_n = 2(t_{n-1} + 2) - (n + 2)$

(D) $t_n = t_{n-1} + 2(n + 1)$

D

Plug in!

3 A quadratic function, f, has zeros P and Q, such that $P + Q = 5$ and $\frac{1}{P} + \frac{1}{Q} = 8$.

Which choice describes f?

(A) $f(x) = 8x^2 - 40x + 5$

B $f(x) = 8x^2 - 40x - 5$

C $f(x) = 2x^2 - 10x + 5$

D $f(x) = 2x^2 - 10x - 5$

A

.128
4.872

Find zeros in calc!

4 Lucy invested $6,000 into an account that earns 6% interest compounded continuously. ***Approximately*** how long will it take for Lucy's investment to be valued at $25,000?

$A = Pe^{Rt}$

A 52.7 years

B 46.9 years

C 24.5 years

(D) 23.8 years

D

$25000 = 6000e^{.06t}$

$\frac{25}{6} = e^{.06t}$

$\log_e \frac{25}{6} = .06t$

$\ln \frac{25}{6} = .06t$

$23.8 = t$

Know formula!

5 A lamppost is located 418 feet from a building. The angle of elevation from the base of the lamppost to the top of the building is 32.3°. ***Approximately*** how tall is the building?

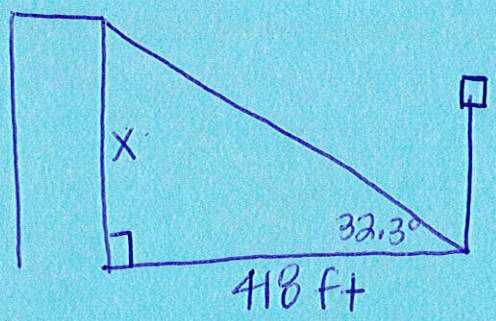
B

A 223 feet

(B) 264 feet

C 510 feet

D 661 feet



$\tan 32.3 = \frac{x}{418}$

$x = 264.2$ ft

Degree mode!

6 Two functions are shown below.

$$T(x) = -x$$
$$P(x) = 10x + 2$$

What is the value of $P(T(3)) - T(P(3))$?

1st 1st

work from inside out!

B

A 8

(B) 4

C 0

D −4

$P(-3) - T(32)$

$-28 - -32$

4

7 A piecewise function is shown below.

$$f(x) = \begin{cases} cx + 1, & x \leq 2 \\ cx^2 - 1, & x > 2 \end{cases}$$

For what value of c does $\lim_{x \to 2} f(x)$ exist?

same value of 2^- & 2^+!

C

A −2

B −1

(C) 1

D 4

8 What are the polar coordinates of (4, 9)?

(R, θ) x y QI

Know formulas!

A

(A) $(\sqrt{97}, 66°)$

B $(\sqrt{97}, 114°)$

~~C~~ $(\sqrt{13}, 66°)$

~~D~~ $(\sqrt{13}, 114°)$

R

$R^2 = x^2 + y^2$

$R^2 = 16 + 81$

$R^2 = 97$

$R = \sqrt{97}$

θ

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{9}{4}$

$\theta = 66.04°$ ✓

Degree mode!

9 A sequence is shown below.

Geometric (use formula sheet!)

$1, 3, 3^2, 3^3, \ldots$

$R = 3$

finite

How many terms of the sequence must be added together for the sum to equal 3,280?

$S_n = \frac{a_1(1-R^n)}{1-R}$ → $3280 = \frac{1-3^n}{-2}$

$3280 = \frac{1(1-3^n)}{1-3}$

$-6560 = 1-3^n$

$-6561 = -3^n$

$6561 = 3^n$ $\quad n = 8$

$\log_3 6561 = n$

C

A 6

B 7

(C) 8

D 9

10 The first term of an infinite geometric sequence is 2. The sum of the sequence is 6. What is the common ratio of the sequence?

use formula sheet!

$S = \frac{a_1}{1-R}$

$6 = \frac{2}{1-R}$

$6 - 6R = 2$ $\quad R = \frac{2}{3}$

$-6R = -4$

B

A $\frac{1}{3}$

(B) $\frac{2}{3}$

C $\frac{3}{3}$

D $\frac{4}{3}$

11 Which is true of the series shown below?

$\pi + \frac{3\pi}{4} + \frac{9\pi}{16} + \frac{27\pi}{64} + \ldots$ $\quad R = \frac{3}{4}$ use formula sheet!

~~A~~ The series diverges. → $|R| > 1$

B The series converges to $\frac{3\pi}{2}$.

C The series converges to $\frac{4\pi}{3}$.

(D) The series converges to 4π.

D

$S = \frac{a_1}{1-R}$

$S = \frac{\pi}{1-\frac{3}{4}} \rightarrow \frac{\pi}{\frac{1}{4}} \rightarrow 4\pi$

12 Karen recursively generated a sequence of five positive integers by starting with a positive integer, a_1, and then applying the recursive formula $a_n = a_{n-1} + 3n - 1$ to generate a_n for $n = 2, 3, 4$, and 5.

If the value of a_5 was 407, what was the value of Karen's starting term, a_1?

$407 = a_4 + 3(5) - 1$

$393 = a_4$

$393 = a_3 + 3(4) - 1$

$382 = a_3$

$382 = a_2 + 3(3) - 1$

$374 = a_2$

$374 = a_1 + 3(2) - 1$

$369 = a_1$ ✓

D

A 366

B 367

C 368

(D) 369

D

13 What is the distance between y-intercepts of the graph of $x + 8 = 2(y + 3)^2$?

parabola opens right

If you want to graph in calc, solve for y!

A (A) 4

B 6

C 11

D 15

y-int (0, -1) & (0, -5)

$\sqrt{\frac{x+8}{2}} = (y+3)^2$

$\pm\sqrt{\frac{x+8}{2}} = y+3$

$y = \pm\sqrt{\frac{x+8}{2}} - 3$

14 Which is a solution set to $x + \frac{3x}{x-1} = \frac{x+2}{x-1}$?

B A $\{-1\}$

(B) $\{-2\}$

C $\{-2, 1\}$

D $\{2, -1\}$

common denom!

$\frac{x(x-1)}{x-1} + \frac{3x}{x-1} = \frac{x+2}{x-1}$

$\frac{x^2 - x + 3x}{x-1} = \frac{x+2}{x-1}$

$x^2 - x + 3x = x + 2$

$x^2 + 2x = x + 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$ ↘ extraneous (÷ by 0)

15 What is the range of the inverse of $y = \tan x$?

A (A) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

B $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

A C $0 < y < \pi$

D $0 \leq y \leq \pi$

$\sin^{-1} = -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\cos^{-1} = 0 \leq y \leq \pi$

$\tan^{-1} = -\frac{\pi}{2} < y < \frac{\pi}{2}$

16 James is standing 10 meters away from Samantha.

- A bird is located in the sky at a point between where James and Samantha are standing.
- James is looking up at the bird at an angle of elevation of 74°.
- Samantha is looking up at the bird at an angle of elevation of 47°.

Approximately how far is the bird from Samantha?

law of sines (use formula sheet!)

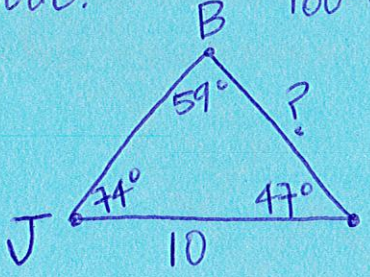
Degree mode!

C A 7.6 meters

B 8.5 meters

(C) 11.2 meters

D 13.1 meters



$180 - 74 - 47 = 59°$

$\frac{\sin 59}{10} = \frac{\sin 74}{x}$

$10 \sin 74 = x \sin 59$

$\frac{10 \sin 74}{\sin 59} = x$

$x = 11.2$ m

inverse of log = exp

17 What is the inverse function of $f(x) = \log_5(2x - 1)$?

A $f^{-1}(x) = 5^x - 1$

B $f^{-1}(x) = \frac{5^x + 1}{2}$

C $f^{-1}(x) = \log_2(5x - 1)$

D $f^{-1}(x) = \log_5 \frac{5x + 1}{2}$

B

$\log_5(2x-1) = y \rightarrow$ switch x & y, solve for y!

$\log_5(2y-1) = x$

$5^x = 2y - 1$

$\frac{5^x + 1}{2} = y$

18 What is the value of the limit shown below?

$$\lim_{n\to\infty}\left(\frac{3^n - 1}{3^n}\right)$$

A $\frac{1}{3}$

B $\frac{2}{3}$

C 1

D $+\infty$

C

horizontal asymptote rules! (degree)

$n = m$

$y = \frac{3}{3}$

$y = 1$

19 What type of conic section is represented by $r = \frac{8}{16 + 125\sin\theta}$?

A circle

B ellipse

C hyperbola

D parabola

C

$\frac{8/16}{\frac{16}{16} + \frac{125}{16}\sin\theta}$

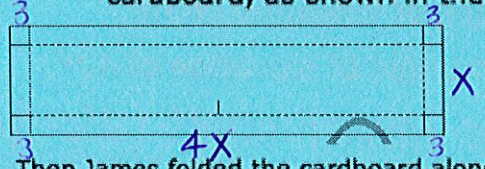
$\frac{.5}{1 + 7.8\sin\theta}$

eccentricity > 1 so hyperbola!

$e = 1$ par.

$e < 1$ ellipse

20 James had a rectangular piece of cardboard that was four times as long as it was wide. He wanted to use the cardboard to make a box with no lid. To do this, he first cut a 3-by-3-inch square out of each of the four corners of the piece of cardboard, as shown in the picture below.



Then James folded the cardboard along the four dotted lines shown in the picture. This created an open box with a volume of 336 cubic inches.

What was the width of the sheet of cardboard that James started with?

A 10.5 inches

B 9.5 inches

C 8.5 inches

D 7.5 inches

B

$l = 4x - 6$

$w = x - 6$

$h = 3$

$V = lwh$

$336 = (4x-6)(x-6)(3)$

solve for x: find zeros in calc!

$x = -\cancel{8}, 9.5$

$\sin^2\theta + \cos^2\theta = 1$ / $1 + \tan^2\theta = \sec^2\theta$ / $1 + \cot^2\theta = \csc^2\theta$

21 Which expression is equivalent to $(\sec\theta)\left(\frac{\sin\theta}{\tan\theta}\right)$?

Identities

$\frac{1}{\cos\theta}\frac{\sin\theta}{\tan\theta} \rightarrow \frac{\tan\theta}{\tan\theta} = 1$

~~A~~ $\cos^2\theta - \sin^2\theta$ Backwards

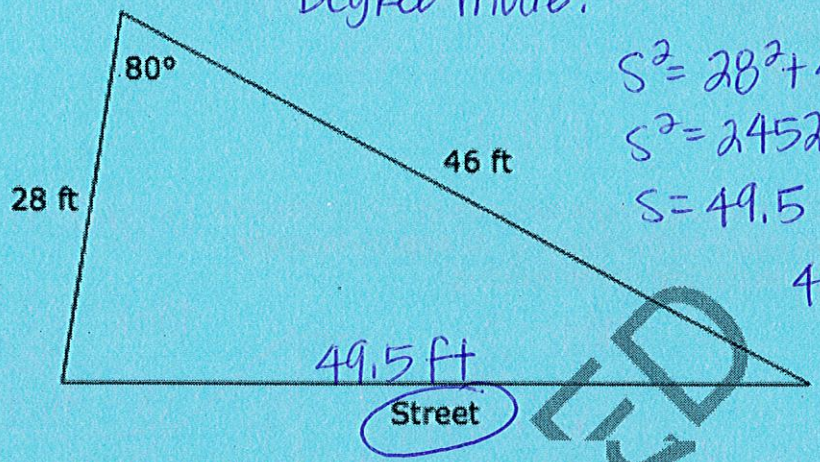
~~B~~ $\sin^2\theta - \cos^2\theta$ addition

D ~~C~~ $\cot^2\theta - \csc^2\theta$ Backwards

(D) $\csc^2\theta - \cot^2\theta$

22 Suppose that for each foot of land along the street, the annual tax is $25 per foot. The diagram below shows a plot of land.

Degree mode! law of cosines (use formula sheet)



$s^2 = 28^2 + 46^2 - 2(28)(46)\cos 80°$

$s^2 = 2452.68$

$s = 49.5$

$49.5(\$25) = \$1,237.5$

RELEASED

About how much is the annual tax for the plot?

(A) $1,238

A B $1,293

C $1,321

D $1,411

23 The function $C(x) = \frac{2.50x + 1.00}{x}$ models the cost per item for a company to produce x items after the first item is made. What is the inverse function of $C(x)$?

$y = \frac{2.50x + 1.00}{x}$ Switch x & y, solve for y!

(A) $C^{-1}(x) = \frac{1.00}{x - 2.50}$

$x = \frac{2.50y + 1.00}{y}$

B $C^{-1}(x) = \frac{x - 2.50}{1.00}$

A $xy = 2.50y + 1.00$

C $C^{-1}(x) = \frac{x - 1.00}{2.50}$

$xy - 2.50y = 1.00$

$y(x - 2.50) = 1.00$

D $C^{-1}(x) = \frac{2.50}{x - 1.00}$

$\frac{y(x-2.50)}{(x-2.50)} = \frac{1.00}{(x-2.50)}$

$y = \frac{1.00}{x - 2.50}$

24 A computer rental company charges $50 to rent a computer for one week. The table below shows the daily late fees the company charges if a computer is returned late.

Days Late	Daily Late Fee
days 1 through 10	$5
days 11 through 20	$8
days 21 through 30	$10

What would be the total cost of renting a computer for one week and returning it 15 days late?

A $120

B $125

C $140

D $170

C

$\$50 + \$5(10 \text{ days}) + \$8(5 \text{ days})$

$50 + 50 + 40$

$140

25 From a point 100 feet from the base of a building, Angie looks up at a 40° angle to the top of a building. She walks 20 feet closer to the building. At ***approximately*** what angle must Angie now look up to see the top of the building?

Degree mode!

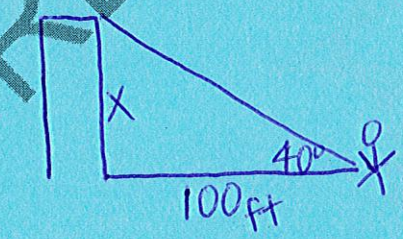
A 32°

B 46°

C 60°

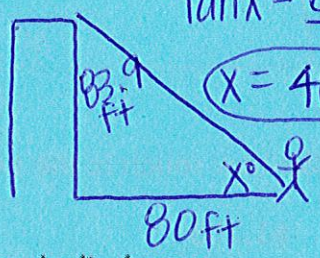
D 77°

B



$\tan 40 = \frac{x}{100}$

$x = 83.9$ ft



$\tan x = \frac{83.9}{80}$

$x = 46.4°$

1 The equation $y = \frac{1}{18}x^2$ represents the mirror inside a parabolic lamp.

$|a| = \frac{1}{4c}$ $\frac{1}{18} = \frac{1}{4c}$ 18

- What is the focal width of the mirror?
- Use the equation to explain your answer.

2 The function $P(t) = 1{,}440e^{-0.0259t}$ models the number of cars a dealership sold t years after the first year it was open.

- By what percent is the number of cars being sold decreasing each year? $R = 2.59\%$
- How many cars did the dealership sell the year it opened? 1,440 initial amount

3 Two parametric equations are shown below.

$x = \frac{3t^2}{2}$ $t = \sqrt{\frac{2x}{3}}$

$y = 4t - 1$ $t = \frac{y+1}{4}$

$x = \frac{3\left(\frac{y+1}{4}\right)^2}{2}$

$y = 4\sqrt{\frac{2x}{3}} - 1$

- Convert the parametric equations into rectangular form.
- Determine what type of equation the rectangular form describes.

square root or parabola