SOLVING SYSTEMS-MATRICES

Gaussian elimination a method use to transform a system of equation into triangular form. Named after Carl Friedrich Gauss (1777-1855)

Augmented Matrix a matrix use to represent a system of linear equations.

$$\begin{cases} 2x - 3y = 8 \\ x + 4y = -7 \end{cases} \Rightarrow \begin{bmatrix} 2 & -3 & 8 \\ 1 & 4 & -7 \end{bmatrix}$$

Row Operations on an augmented matrix are any of the following:

1. switch two rows
2. multiply a row by a constant
3. add one row to another

Row Echelon Form using row operations to obtain triangulation of the matrix. When the matrix is in reduced row echelon form the solutions are obtained.

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ x & y & z & \end{bmatrix} \quad \begin{matrix} x=6 \\ y=-2 \\ z=3 \end{matrix} \quad (6, -2, 3)$$

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An Augmented Matrix

EX #1: Write the augmented matrix.

A. $\begin{cases} 5x + 4y = 7 \\ 4x - 2y = 16 \end{cases}$ $\begin{bmatrix} 5 & 4 & 7 \\ 4 & -2 & 16 \end{bmatrix}$

B. $\begin{cases} x + 3y - 2z = 9 \\ 2x \quad\quad - 6z = 2 \\ \quad 2y - 2z = 8 \end{cases}$ $\begin{bmatrix} 1 & 3 & -2 & 9 \\ 2 & 0 & -6 & 2 \\ 0 & 2 & -2 & 8 \end{bmatrix}$

EX #2: Write the system of linear equations represented by the augmented matrix. Use variables *x*, *y*, and *z*.

A. $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 3 & -2 & 0 & 2 \\ 5 & 3 & -1 & 1 \end{bmatrix}$ $\begin{cases} x + y + z = 2 \\ 3x - 2y \quad\quad = 2 \\ 5x + 3y - z = 1 \end{cases}$

B. $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -2 & 0 & 9 \\ 0 & 3 & 4 & 6 \end{bmatrix}$ $\begin{cases} x + y + z = 6 \\ x - 2y \quad\quad = 9 \\ \quad\quad 3y + 4z = 6 \end{cases}$

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Obj: I will be able to:

1. Solve systems using Gaussian Elimination, reduced row echelon form, and augmented matrix.

Applying Row Operations on a Matrix

ROW OPERATIONS ON A MATRIX

1. Interchange any two rows.
2. Replace a row by a nonzero constant multiple of that row.
3. Replace a row by the sum of that row and a nonzero constant multiple of some other row.

EX #3: Using the matrix below, find a row operation for the following: $\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 3 & 4 & 7 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & - \\ 0 & 1 & - \end{array}\right]$

A. Having a 0 in row 1, column 2.

$-2R_1 + R_2 = R_1$

$-2R_1 \quad -2 \quad -4 \quad -10$

$\quad\quad 3 \quad\quad 4 \quad\quad 7$

$1 \quad 0 \quad -3 \quad R_1$

$\left[\begin{array}{ccc} 1 & 0 & -3 \\ 3 & 4 & 7 \end{array}\right]$

B. Having a 0 in row 2, column 1.

$-3R_1 + R_2 = R_2$

$-3 \quad 0 \quad 9$

$3 \quad 4 \quad 7$

$0 \quad 4 \quad 16$

$\left[\begin{array}{ccc} 1 & 0 & -3 \\ 0 & 4 & 16 \end{array}\right]$

C. Having a 1 in row 2, column 2.

$\frac{1}{4}R_2 = R_2$

$0 \quad 1 \quad 4$

$\left[\begin{array}{ccc} 1 & 0 & -3 \\ 0 & 1 & 4 \end{array}\right]$ (x y)

$(-3, 4)$ $\quad x = -3 \quad y = 4$

CCSS: HSN.VM.C.6; HSN.VM.C.7; HSN.VM.C.8; HSN.VM.C.9; HSN.VM.C.10

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Solving a System Using Matrices

EX #4: Given $\begin{cases} x+y-2z=6 \\ y+3z=-8 \\ z=-3 \end{cases}$

A. Write the augmented matrix.

$$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 3 & -8 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

B. Perform the indicated row operations. State the solution.

$R_2 = -3r_3 + r_2$	$R_1 = r_1 - r_2$	$R_1 = 2r_3 + r_1$
$\begin{matrix} 0 & 0 & -3 & 9 \\ + & 0 & 1 & 3 & -8 \end{matrix}$	$\begin{matrix} 1 & 1 & -2 & 6 \\ -(0 & 1 & 0 & 1) \end{matrix}$	$\begin{matrix} 0 & 0 & 2 & -6 \\ 1 & 0 & -2 & 5 \end{matrix}$
$\begin{matrix} 0 & 1 & 0 & 1 \end{matrix}$ R_2	$\begin{matrix} 1 & 0 & -2 & 5 \end{matrix}$ R_1	$\begin{matrix} 1 & 0 & 0 & -1 \end{matrix}$ R_1
$\begin{bmatrix} 1 & 1 & -2 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

$x = -1 \quad y = 1 \quad z = -3$

$(-1, 1, -3)$

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