SECTION 2.3

Exploration 1

1. **(a)** ∞; $-\infty$ **(b)** $-\infty$; ∞ **(c)** ∞; $-\infty$ **(d)** $-\infty$; ∞ **3.** **(a)** $-\infty$; ∞ **(b)** $-\infty$; $-\infty$ **(c)** ∞; ∞ **(d)** ∞; $-\infty$

Exploration 2

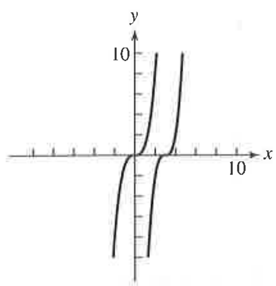
1. $y = 0.0061x^3 + 0.0177x^2 - 0.5007x + 0.9769$

Quick Review 2.3

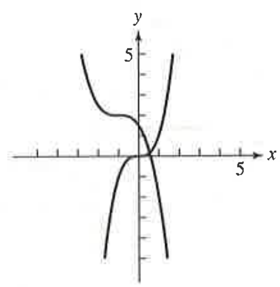
1. $(x - 4)(x + 3)$ **3.** $(3x - 2)(x - 3)$ **5.** $x(3x - 2)(x - 1)$ **7.** $x = 0, x = 1$ **9.** $x = -6, x = -3, x = 1.5$

Exercises 2.3

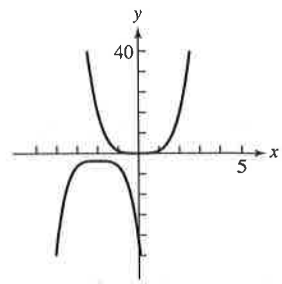
1. Shift $y = x^3$ to the right by 3 units, stretch vertically by 2. y-intercept: $(0, -54)$



3. Shift $y = x^3$ to the left by 1 unit, vertically shrink by $\frac{1}{2}$, reflect over the x-axis, and then vertically shift up 2 units. y-intercept: $\left(0, \frac{3}{2}\right)$

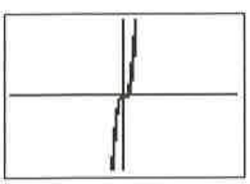


5. Shift $y = x^4$ to the left 2 units, vertically stretch by 2, reflect over the x-axis, and vertically shift down 3 units. y-intercept: $(0, -35)$



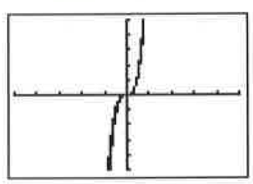
7. local maximum: $\approx (0.79, 1.19)$, zeros: $x = 0$ and $x \approx 1.26$. **9.** (c) **11.** (a)

13. One possibility:



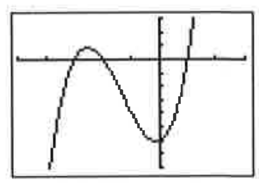
$[-100, 100]$ by $[-1000, 1000]$

15. One possibility:



$[-50, 50]$ by $[-1000, 1000]$

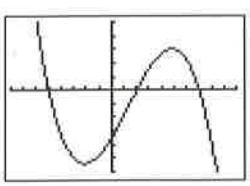
17.



$[-5, 3]$ by $[-8, 3]$

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = -\infty$

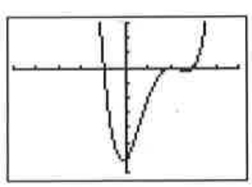
19.



$[-8, 10]$ by $[-120, 100]$

$\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = \infty$

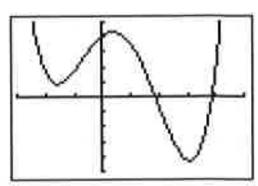
21.



$[-5, 5]$ by $[-14, 6]$

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = \infty$

23.

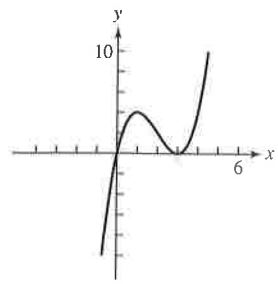


$[-3, 5]$ by $[-50, 50]$

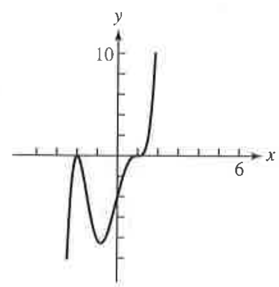
$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = \infty$

25. ∞, ∞ **27.** $-\infty, \infty$ **29.** (a) There are 3 zeros: they are -2.5, 1, and 1.1. **31.** (c) There are 3 zeros: approximately -0.273 (actually $-3/11$), -0.25, and 1. **33.** -4 and 2 **35.** 2/3 and $-1/3$ **37.** 0, $-2/3$, and 1

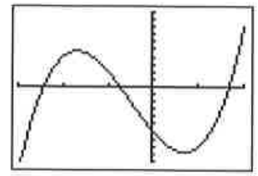
39. Degree 3; zeros: $x = 0$ (mult. 1, graph crosses x-axis), $x = 3$ (mult. 2, graph is tangent)



41. Degree 5; zeros: $x = 1$ mult. 3, graph crosses x-axis), $x = -2$ (mult. 2, graph is tangent)



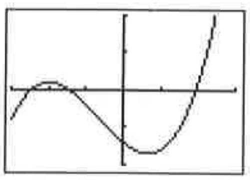
43.



$[-3, 2]$ by $[-10, 10]$

$-2.43, -0.74, 1.67$

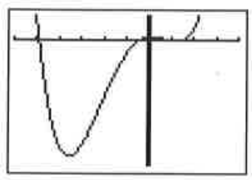
45.



$[-3, 3]$ by $[-10, 10]$

$-2.47, -1.46, 1.94$

47.



$[-6, 4]$ by $[-100, 20]$

$-4.90, -0.45, 1, 1.35$

49. 0, −6, and 6 **51.**. −5, 1, 11

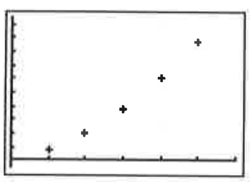
53. $f(x) = x^3 - 5x^2 - 18x + 72$

55. $f(x) = x^3 - 4x^2 - 3x + 12$

57. $y = 0.25x^3 - 1.25x^2 - 6.75x + 19.75$

59. $y = -2.21x^4 + 45.75x^3 - 339.79x^2 + 1075.25x - 1231$ **61.** It follows from the Intermediate Value Theorem.

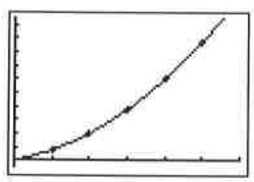
63. (a)



$[0, 60]$ by $[-10, 210]$

(b) $y = 0.051x^2 + 0.97x + 0.26$

(c)

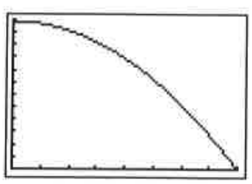


$[0, 60]$ by $[-10, 210]$

(d) ≈ 56.39 ft

(e) 67.74 mph

65. (a)



$[0, 0.8]$ by $[0, 1.20]$

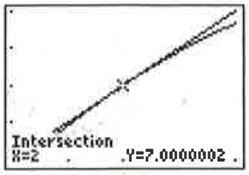
(b) 0.3391 cm

67. $0 < x \leq 0.929$ or $3.644 \leq x < 5$ **69.** True. Because f is continuous and $f(1) = -2$ and $f(2) = 2$, the Intermediate Value Theorem assures us that the graph of f crosses the x-axis between $x = 1$ and $x = 2$. **71.** (c) **73.** (b)

77. The exact behavior near $x = 1$ is hard to see. A zoomed–in view around the point (1, 0) suggests that the graph just touches the x-axis at 0 without actually crossing it — that is, (1, 0) is a local maximum. One possible window is $[0.9999, 1.0001]$ by $[-1 \times 10^{-7}, 1 \times 10^{-7}]$. **79.** A maximum and minimum are not visible in the standard window, but can be seen on the window $[0.2, 0.4]$ by $[5.29, 5.3]$. **81.** The graph of $y = 3(x^3 - x)$ increases, then decreases, then increases; the graph of $y = x^3$ only increases. Therefore, this graph can not be obtained from the graph of $y = x^3$ by the transformations studied in Chapter 1 (translations, reflections, and stretching/shrinking). Since the right side includes only these transformations, there can be no solution.

83. (a) Substituting $x = 2$, $y = 7$, we find that $7 = 5(2 - 2) + 7$, so Q is on line L, and also $f(2) = -8 + 8 + 18 - 11 = 7$, so Q is on the graph of $f(x)$.

(b)



[1.8, 2.2] by [6, 8]

(c) The line L also crosses the graph of $f(x)$ at $(-2, -13)$.

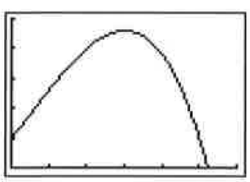
SECTION 2.4

Quick Review 2.4

1. $x^2 - 4x + 7$ **3.** $7x^3 + x^2 - 3$ **5.** $x(x + 2)(x - 2)$ **7.** $4(x + 5)(x - 3)$ **9.** $(x + 2)(x + 1)(x - 1)$

Exercises 2.4

1. $f(x) = (x - 1)^2 + 2; \frac{f(x)}{x - 1} = x - 1 + \frac{2}{x - 1}$ **3.** $f(x) = (x^2 + x + 4)(x + 3) - 21; \frac{f(x)}{x + 3} = x^2 + x + 4 - \frac{21}{x + 3}$
5. $f(x) = (x^2 - 4x + 12)(x^2 + 2x - 1) - 32x + 18; \frac{f(x)}{x^2 + 2x - 1} = x^2 - 4x + 12 + \frac{-32x + 18}{x^2 + 2x - 1}$
7. $x^2 - 6x + 9 + \frac{-11}{x + 1}$ **9.** $9x^2 + 97x + 967 + \frac{9670}{x - 10}$ **11.** $-5x^3 - 20x^2 - 80x - 317 + \frac{1269}{4 - x}$ **13.** 3 **15.** -43
17. 5 **19.** Yes **21.** No **23.** Yes **25.** $f(x) = (x + 3)(x - 1)(5x - 17)$ **27.** $2x^3 - 6x^2 - 12x + 16$
29. $2x^3 - 8x^2 + \frac{19}{2}x - 3$ **31.** $f(x) = 3(x + 4)(x - 3)(x - 5)$ **33.** $\frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$; 1 **35.** $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$; $\frac{3}{2}$
45. No zeros outside window. **47.** There *are* zeros not shown (approx. -11.002 and 12.003).
49. Rational zero: $\frac{3}{2}$; irrational zeros: $\pm\sqrt{2}$ **51.** Rational: -3; irrational: $1 \pm \sqrt{3}$ **53.** Rational: -1 and 4; irrational: $\pm\sqrt{2}$ **55.** Rational: $-\frac{1}{2}$ and 4; irrational: none **57.** \$36.27; 53.7 **59.** -2 **61. (b)** 2 is a zero of $f(x)$.
(c) $(x - 2)(x^3 + 4x^2 - 3x - 19)$ **(d)** One irrational zero is $x \approx 2.04$. **(e)** $f(x) \approx (x - 2)(x - 2.04)(x^2 + 6.04x + 9.3216)$
63. False. $(x + 2)$ is a factor if and only if $f(-2) = 0$. **65.** (a) **67.** (b) **69. (d)** $x \approx 0.6527$ m
71. (a) Shown is one possible view, on the window [0, 600] by [0, 500].


[0, 600] by [0, 500]

(b) The maximum population, after 300 days, is 460 turkeys.
(c) $P = 0$ when $t \approx 523.22$ — about 523 days after release.

73. (a) 0 or 2 positive zeros, 1 negative zero **(b)** no positive zeros, 1 or 3 negative zeros **(c)** 1 positive zero, no negative zeros **(d)** 1 positive zero, 1 negative zero **77. (b)** zeros: $-\frac{7}{3}, \frac{1}{2}, 3$ **(c)** There are no rational zeros.
79. (a) Approximate zeros: $-3.126, -1.075, 0.910, 2.291$. **(b)** $f(x) \approx g(x) = (x + 3.126)(x + 1.075)(x - 0.910)(x - 2.291)$
(c) Graphically: graph the original function and the approximate factorization on a variety of windows and observe their similarity. Numerically: Compute $f(c)$ and $g(c)$ for several values of c.

SECTION 2.5

Exploration 1

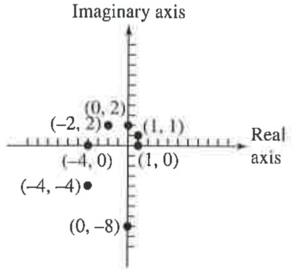
1. $\left\{1, -\frac{1}{2} + \frac{i\sqrt{3}}{2}, -\frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}$ exactly one unit

Quick Review 2.5

1. $x + 9$ **3.** $a + 2d$ **5.** $x^2 - x - 6$ **7.** $x^2 - 2$ **9.** $x^2 - 2x - 1$

Exercises 2.5

1. $8 + 2i$ **3.** $13 - 4i$ **5.** $5 - (1 + \sqrt{3})i$ **7.** $-5 + i$ **9.** $7 + 4i$ **11.** $-5 - 14i$ **13.** $-48 - 4i$ **15.** $5 - 10i$
17. $4i$ **19.** $\sqrt{3}i$ **21.** $x = 2, y = 3$ **23.** $x = 1, y = 2$ **25.** $5 + 12i$ **27.** -1 **29.** 13 **31.** 25 **33.** $\frac{2}{5} - \frac{1}{5}i$
35. $\frac{3}{5} + \frac{4}{5}i$ **37.** $\frac{1}{2} - \frac{7}{2}i$ **39.** $\frac{7}{5} - \frac{1}{5}i$ **41.** (f) **43.** (c) **45.** $x = -1 \pm 2i$ **47.** $x = \frac{7}{8} \pm \frac{\sqrt{15}}{8}i$
49. length $= 8$, midpoint: $(2, 0)$ **51.** length $= \sqrt{13}$, midpoint: $-\frac{1}{2} - 5i$ **55.** False. Any complex number bi has this property.
57. (e) **59.** (a) **61.** **(a)** $i, -1, -i, 1, i, -1, -i, 1$ **(b)** $-i, -1, i, 1, -i, -1, i, 1$ **(c)** 1
63. **(a)** real $= 8$, imaginary $= -8$; real $= 16$, imaginary $= 0$.
(b)



(c) Answers will vary. **(d)** $(1 + i)^7 = 8 - 8i$, $(1 + i)^8 = 16$
67. (d), (h), (e), (c), (f), (b), (g), (a)
69. **(a)** Not always true. **(b)** Always true. **(c)** Not always true.

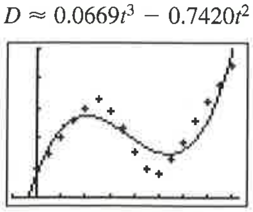
SECTION 2.6

Quick Review 2.6

1. $(2x - 3)(x + 1)$ **3.** $(5x - 4)(2x + 1)$ **5.** $\frac{5}{2} \pm \frac{\sqrt{19}}{2}i$ **7.** $\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}$ **9.** $\pm 1, \pm 2, \pm 4, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}$

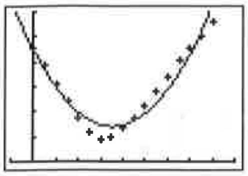
Exercises 2.5

1. $x^2 + 9$; zeros: $\pm 3i$; x-intercepts: none **3.** $x^4 - 2x^3 + 5x^2 - 8x + 4$; zeros: 1 (mult. 2), $\pm 2i$; x-intercepts: $x = 1$
5. $x^2 + 1$ **7.** $x^3 - x^2 + 9x - 9$ **9.** $x^4 - 5x^3 + 7x^2 - 5x + 6$ **11.** $x^3 - 11x^2 + 43x - 65$
13. $x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$ **15.** $x^4 - 10x^3 + 38x^2 - 64x + 40$ **17.** (b) **19.** (d)
21. 2 complex zeros; none real **23.** 3 complex zeros; 1 real **25.** 4 complex zeros; 2 real
27. Zeros: $x = 1, x = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i$ $f(x) = \frac{1}{4}(x - 1)(2x + 1 + \sqrt{19}i)\ (2x + 1 - \sqrt{19}i)$
29. Zeros: $x = \pm 1, x = -\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$ $f(x) = \frac{1}{4}(x - 1)(x + 1)(2x + 1 + \sqrt{23}i)\ (2x + 1 - \sqrt{23}i)$
31. Zeros: $x = -\frac{7}{3}, x = \frac{3}{2}, x = 1 \pm 2i$ $f(x) = (3x + 7)(2x - 3)(x - 1 + 2i)(x - 1 - 2i)$
33. Zeros: $x = \pm\sqrt{3}, x = 1 \pm i$ $f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 1 + i)\ (x - 1 - i)$
35. Zeros: $x = \pm 2, x = 3 \pm 2i$ $f(x) = (x - \sqrt{2})(x + \sqrt{2})(x - 3 + 2i)\ (x - 3 - 2i)$ **37.** $f(x) = (x - 2)(x^2 + x + 1)$
39. $f(x) = (x - 1)(2x^2 + x + 3)$ **41.** $f(x) = (x - 1)(x + 4)(x^2 + 1)$ **43.** $h \approx 3.776$ ft
49. $f(x) = -2x^4 + 12x^3 - 20x^2 - 4x + 30$ **51.** False. If $1 - 2i$ is a zero then $1 + 2i$ must also be a zero. **53.** (e) **55.** (c)
57. **(a)** $D \approx 0.0669t^3 - 0.7420t^2 + 2.1759t + 0.8250$ **(b)** 0.825 m **(c)** $t \approx 2.02$ sec ($D \approx 2.74$ m) and $t \approx 5.38$ sec ($D \approx 1.47$ m)



[−1, 8.25] by [0, 5]

59. **(a)** $D \approx 0.2434t^2 - 1.7159t + 4.4241$ **(b)** Jacob walks toward the detector, then turns and walks away (or walks backwards). **(c)** The model "changes direction" at $t \approx 3.52$ (when $D \approx 1.40$ m).



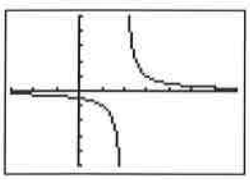
[−1, 9] by [0, 6]

61. $f(i) = i^3 - i(i)^2 + 2i(i) + 2 = -i + i - 2 + 2 = 0$ **63.** Synthetic division shows that $f(i) = 0$ (the remainder), and at the same time gives $f(x) \div (x - i) = x^2 + 3x - i = h(x)$, so $f(x) = (x - i)(x^2 + 3x - i)$. **65.** $4 - 2i, 2 + 6i$
67. $-4, 2 + 2\sqrt{3}i, 2 - 2\sqrt{3}i$

SECTION 2.7

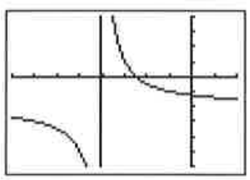
Exploration 1

1. $g(x) = \dfrac{1}{x - 2}$



[−3, 7] by [−5, 5]

3. $k(x) = \dfrac{3}{x + 4} - 2$



[−8, 2] by [−5, 5]

Quick Review 2.7

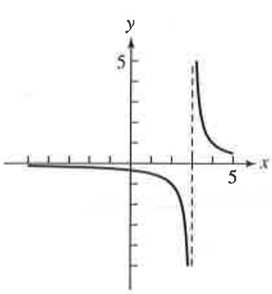
1. $x = -3$ or $x = \dfrac{1}{2}$ **3.** $x = \pm 2$ **5.** $x = 1$ **7.** 2; 7 **9.** 3; −5

Exercises 2.7

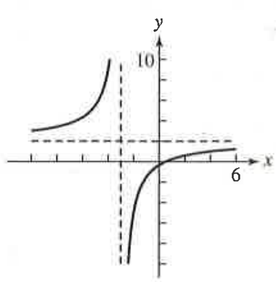
1. Domain: all $x \neq -3$; $\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = \infty$

3. Domain: all $x \neq -2, 2$; $\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -2^+} f(x) = \infty$, $\lim_{x \to 2^-} f(x) = \infty$, $\lim_{x \to 2^+} f(x) = \infty$

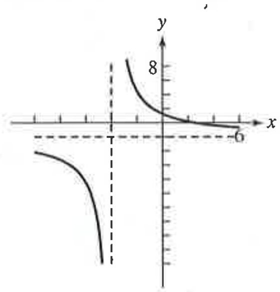
5. Translate right 3 units. Asymptotes: $x = 3$, $y = 0$



7. Translate left 3 units, reflect across x-axis, vertically stretch by 7, translate up 2 units. Asymptotes: $x = -3$, $y = 2$



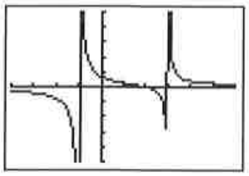
9. Translate left 4 units, vertically stretch by 13, translate down 2 units. Asymptotes: $x = -4$, $y = -2$



11. ∞ **13.** 0 **15.** ∞ **17.** 5 **19.** Vertical asymptotes: none; Horizontal asymptote: $y = 2$; $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 2$
21. Vertical asymptotes: $x = 0$, $x = 1$; Horizontal asymptote: $y = 0$; $\lim_{x \to 0^-} f(x) = \infty$, $\lim_{x \to 0^+} f(x) = -\infty$, $\lim_{x \to 1^-} f(x) = -\infty$, $\lim_{x \to 1^+} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$

23. Intercepts: $\left(0, \frac{2}{3}\right)$ and $(2, 0)$

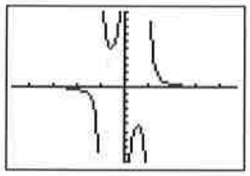
Asymptotes: $x = -1$, $x = 3$, and $y = 0$



$[-4, 6]$ by $[-5, 5]$

25. No intercepts

Asymptotes: $x = -1$, $x = 0$, $x = 1$, and $y = 0$



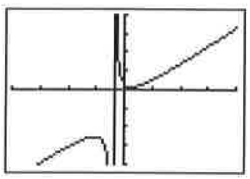
$[-4.7, 4.7]$ by $[-10, 10]$

27. Intercepts: $(0, 2)$, $(-1.28, 0)$, and $(0.78, 0)$; Asymptotes: $x = 1$, $x = -1$, and $y = 2$

$[-5, 5]$ by $[-4, 6]$

29. Intercept: $\left(0, \frac{3}{2}\right)$

Asymptotes: $x = -2$, $y = x - 4$



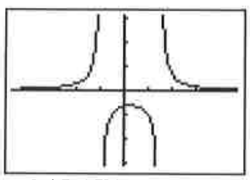
$[-20, 20]$ by $[-20, 20]$

31. (d); Xmin = −2, Xmax = 8, Xscl = 1, and Ymin = −3, Ymax = 3, Yscl = 1

33. (a); Xmin = −3, Xmax = 5, Xscl = 1, and Ymin = −5, Ymax = 10, Yscl = 1

35. (e); Xmin = −2, Xmax = 8, Xscl = 1, and Ymin = −3, Ymax = 3, Yscl = 1

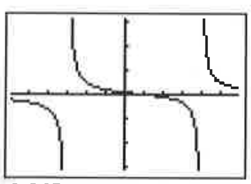
37. Intercept: $\left(0, -\frac{2}{3}\right)$; asymptotes: $x = -1$, $x = \frac{3}{2}$, $y = 0$; $\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to (3/2)^-} f(x) = -\infty$, $\lim_{x \to (3/2)^+} f(x) = \infty$;



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $x \neq -1, \frac{3}{2}$; Range: $\left(-\infty, -\frac{16}{25}\right] \cup (0, \infty)$; Continuity: all $x \neq -1, \frac{3}{2}$; Increasing: $(-\infty, -1)$, $\left(-1, \frac{1}{4}\right]$, Decreasing: $\left[\frac{1}{4}, \frac{3}{2}\right)$, $\left(\frac{3}{2}, \infty\right)$; Unbounded; Local Maximum at $\left(\frac{1}{4}, -\frac{16}{25}\right)$; Horizontal asymptote: $y = 0$; Vertical asymptotes: $x = -1$, $x = \frac{3}{2}$; End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$

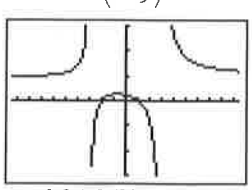
39. Intercepts: $\left(0, \frac{1}{12}\right)$, $(1, 0)$; asymptotes: $x = -3$, $x = 4$, $y = 0$; $\lim_{x \to -3^-} h(x) = -\infty$, $\lim_{x \to -3^+} h(x) = \infty$, $\lim_{x \to 4^-} h(x) = -\infty$, $\lim_{x \to 4^+} h(x) = \infty$



$[-5.875, 5.875]$ by $[-3.1, 3.1]$

Domain: $x \neq -3, 4$; Range: $(-\infty, \infty)$; Continuity: all $x \neq -3, 4$; Decreasing: $(-\infty, -3)$, $(-3, 4)$, $(4, \infty)$; No symmetry; Unbounded; No extrema; Horizontal asymptote: $y = 0$; Vertical asymptotes: $x = -3$, $x = 4$; End behavior: $\lim_{x \to -\infty} h(x) = \lim_{x \to \infty} h(x) = 0$

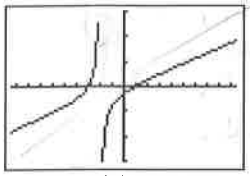
41. Intercepts: $(-2, 0)$, $(1, 0)$, $\left(0, \frac{2}{9}\right)$; asymptotes: $x = -3$, $x = 3$, $y = 1$; $\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = -\infty$, $\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$



$[-9.4, 9.4]$ by $[-3, 3]$

Domain: $x \neq -3, 3$; Range: $(-\infty, 0.260) \cup (1, \infty)$; Continuity: all $x \neq -3, 3$; Increasing: $(-\infty, -3)$, $(-3, -0.675)$; Decreasing: $(-0.675, 3)$, $(3, \infty)$; No symmetry; Unbounded; Local maximum at $(-0.675, 0.260)$; Horizontal asymptote: $y = 1$; Vertical asymptotes: $x = -3$, $x = 3$; End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 1$

43. Intercepts: $(-3, 0)$, $(1, 0)$, $\left(0, -\frac{3}{2}\right)$; asymptotes: $x = -2$, $y = x$; $\lim_{x \to -2^-} h(x) = \infty$, $\lim_{x \to -2^+} h(x) = -\infty$

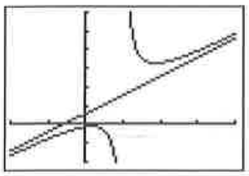


[–9.4, 9.4] by [–15, 15]

Domain: $x \neq -2$, Range: $(-\infty, \infty)$;
Continuity: all $x \neq -2$;
Increasing: $(-\infty, -2)$, $(-2, \infty)$;
No symmetry; Unbounded; No extrema;
Horizontal asymptote: none; Vertical asymptote: $x = -2$;
Slant asymptote: $y = x$; End behavior: $\lim_{x \to -\infty} h(x) = -\infty$, $\lim_{x \to \infty} h(x) = \infty$

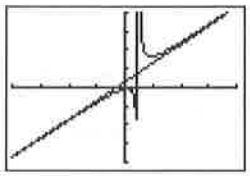
45. $y = x + 3$

(a)



[–10, 20] by [–10, 30]

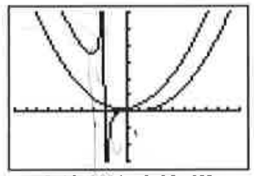
(b)



[–40, 40] by [–40, 40]

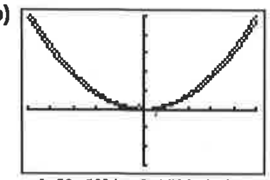
47. $y = x^2 - 3x + 6$

(a)



[–10, 10] by [–30, 60]

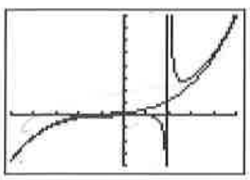
(b)



[–50, 50] by [–1500, 2500]

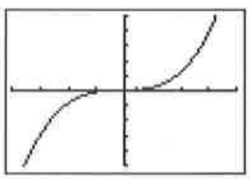
49. $y = x^3 + 2x^2 + 2x + 4$

(a)



[–5, 5] by [–100, 200]

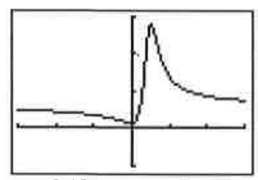
(b)



[–20, 20] by [–5000, 5000]

51. Intercepts: $\left(0, \frac{4}{5}\right)$; Domain: $(-\infty, \infty)$; Range: $[0.773, 14.227]$; Continuity: $(-\infty, \infty)$;

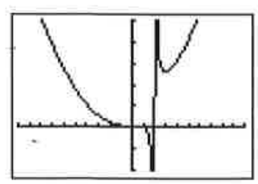
Increasing: $[-0.245, 2.445]$; Decreasing: $(-\infty, -0.245]$, $[2.445, \infty)$; No symmetry;
Bounded; Local max at $(2.445, 14.227)$, local min at $(-0.245, 0.773)$; Horizontal asymptote: $y = 3$;
Vertical asymptote: none; End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 3$



[–15, 15] by [–5, 15]

53. Intercepts: $(1, 0)$, $\left(0, \frac{1}{2}\right)$; Domain: $(x \neq 2)$; Range: $(-\infty, \infty)$; Continuity: $x \neq 2$;

Increasing: $[-0.384, 0.442]$, $[2.942, \infty)$; Decreasing: $(-\infty, -0.384]$, $[0.442, 2)$, $(2, 2.942]$;
No symmetry; Not bounded; Local max at $(0.442, 0.586)$, local min at $(-0.384, 0.443)$
and $(2.942, 25.970)$; Horizontal asymptote: none; Vertical asymptote: $x = 2$;
End behavior $\lim_{x \to -\infty} h(x) = \lim_{x \to \infty} h(x) = \infty$; End behavior asymptote: $y = x^2 + 2x + 4$



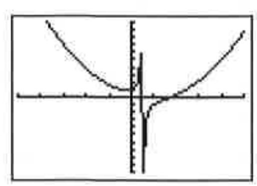
[–10, 10] by [–20, 50]

55. Intercepts: $(1.755, 0)$, $(0, 1)$; Domain: $x \neq \frac{1}{2}$; Range: $(-\infty, \infty)$;

Increasing: $[-0.184, 1/2]$, $\left(\frac{1}{2}, \infty\right)$; Decreasing: $(-\infty, -0.184]$; Not symmetric; Not bounded;

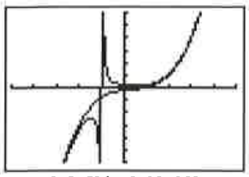
Local min at $(-0.184, 0.920)$; Horizontal asymptote: none; Vertical asymptotes: $x = \frac{1}{2}$;

End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$; End behavior symptote: $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$



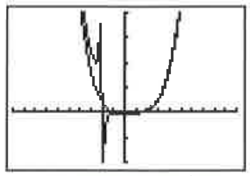
[–5, 5] by [–10, 10]

57. Intercept: (0, 1);
Asymptote: $x = -1$;
End behavior asymptote:
$y = x^3 - x^2 + x - 1$



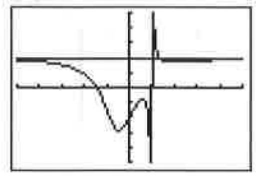
[–5, 5] by [–30, 30]

59. Intercepts: $(1, 0)$, $\left(0, -\frac{1}{2}\right)$;
Asymptote: $x = -2$;
End behavior asymptote:
$y = x^4 - 2x^3 + 4x^2 - 8x + 16$



[–10, 10] by [–200, 400]

61. Intercepts: $(-1.476, 0)$, $(0, -2)$;
Asymptote: $x = 1$;
End behavior asymptote: $y = 2$



[–5, 5] by [–5, 5]

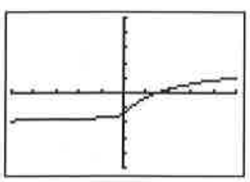
63. False. $1/(x^2 + 1)$ is a rational function and has no vertical asymptotes. **65.** (e) **67.** (d)

69. **(a)** No. The domain of f is $(-\infty, 3) \cup (3, \infty)$; the domain of g is all real numbers. **(b)** No. While it is not defined at 3, it does not tend toward $\pm\infty$ on either side. **(c)** Most grapher viewing windows do not reveal that f is undefined at 3. **(d)** Almost—but not quite; they are equal for all $x \neq 3$.

71. **(b)** If $f(x) = kx^a$, where a is a negative integer, then the power function f is also a rational function. **(c)** 4.22 L

73. Horizontal asymptotes: $y = -2$ and $y = 2$.
Intercepts: $\left(0, -\frac{3}{2}\right)$, $\left(\frac{3}{2}, 0\right)$

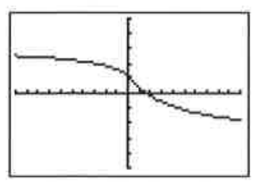
$$h(x) = \begin{cases} \dfrac{2x - 3}{x + 2} & x \geq 0 \\ \dfrac{2x - 3}{-x + 2} & x < 0 \end{cases}$$



[–5, 5] by [–5, 5]

75. Horizontal asymptotes: $y = \pm 3$.
Intercepts: $\left(0, \frac{5}{4}\right)$, $\left(\frac{5}{3}, 0\right)$

$$f(x) = \begin{cases} \dfrac{5 - 3x}{x + 4} & x \geq 0 \\ \dfrac{5 - 3x}{-x + 4} & x < 0 \end{cases}$$



[–10, 10] by [–5, 5]

SECTION 2.8

Quick Review 2.8

1. $2x^2 + 8x$ **3.** LCD: 36; $-\frac{1}{36}$ **5.** LCD: $(2x + 1)(x - 3)$; $\frac{x^2 - 7x - 2}{(2x + 1)(x - 3)}$ **7.** $\frac{3 \pm \sqrt{17}}{4}$ **9.** $\frac{-1 \pm \sqrt{7}}{3}$

Exercises 2.8

1. $x = -1$ **3.** $x = 2$ or $x = -7$ **5.** $x = -4$ or $x = 3$, the latter is extraneous. **7.** $x = 2$ or $x = 5$

9. $x = 3$ or $x = 4$ **11.** $x = \frac{1}{2}$ or $x = -1$, the latter is extraneous. **13.** $x = -\frac{1}{3}$ or $x = 2$, the latter is extraneous.

15. $x = 5$ or $x = 0$, the latter is extraneous. **17.** $x = -2$ or $x = 0$, both of these are extraneous (there are no real solutions).

19. $x = -2$ **21.** Both **23.** $x = 3 + \sqrt{2} \approx 4.414$ or $x = 3 - \sqrt{2} \approx 1.586$ **25.** $x = 1$ **27.** No real solutions

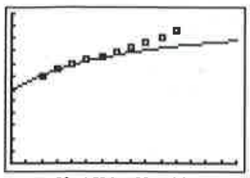
29. $x \approx -3.100$ or $x \approx 0.661$ or $x \approx 2.439$ **31.** **(a)** The total amount of solution is $(125 + x)$ mL; of this, the amount of acid is x plus 60% of the original amount, or $x + 0.6(125)$. **(c)** $C(x) = \frac{x + 75}{x + 125} = 0.83x \approx 169.12$ mL. **33.** **(a)** $C(x) = \frac{3000 + 2.12x}{x}$

(b) 4762 hats per week **(c)** 6350 hats **35.** **(a)** $P(x) = 2x + \frac{364}{x}$ **(b)** $x \approx 13.49$ (a square); $P \approx 53.96$

37. **(a)** $S = 2\pi x^2 + \frac{1000}{x}$ **(b)** Either $x \approx 1.12$ cm and $h \approx 126.66$ cm or $x \approx 11.37$ cm and $h \approx 1.23$ cm

39. **(a)** $R(x) = \frac{2.3x}{x + 2.3}$ **(b)** $x \approx 6.52$ ohms **41.** **(a)** $D(t) = \frac{4.75 + t}{4.75t}$ **(b)** $t \approx 5.74$ h

43. **(a)**



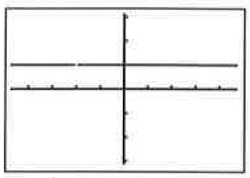
[0, 15] by [0, 120]

(b) About 98.3 billion dollars

45. False. An extraneous solution is a solution of the equation cleared of fractions that is *not* a solution of the original equation.

47. (d) **49.** (e) **51.** **(a)** $f(x) = \frac{x^2 + 2x}{x^2 + 2x}$ **(b)** $x \neq 0, -2$ **(c)** $f(x) = \begin{cases} 1, & x \neq -2, 0 \\ \text{undefined}, & x = -2 \text{ or } x = 0 \end{cases}$

(d) The graph appears to be the horizontal line $y = 1$ with holes at $x = -2$ and $x = 0$.



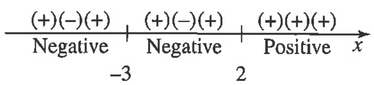
[−4.7, 4.7] by [−3.1, 3.1]

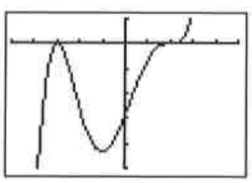
53. $x = \frac{y}{y - 1}$ **55.** $x = \frac{2y - 3}{y - 2}$

SECTION 2.9

Exploration 1

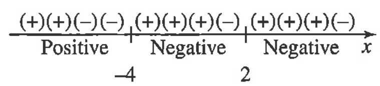
1.

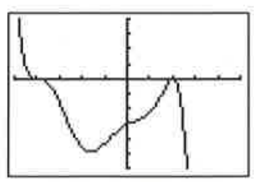




[−5, 5] by [−250, 50]

3.





[−5, 5] by [−3000, 2000]

Quick Review 2.9

1. $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = -\infty$ **3.** $\lim_{x\to\infty} g(x) = \infty$, $\lim_{x\to-\infty} g(x) = \infty$ **5.** $\frac{x^3 + 5}{x}$ **7.** $\frac{x^2 - 7x - 2}{2x^2 - 5x - 3}$

9. **(a)** $\pm 1, \pm\frac{1}{2}, \pm 3, \pm\frac{3}{2}$ **(b)** $(x + 1)(2x - 3)(x + 1)$

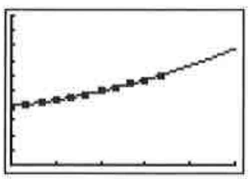
Exercises 2.9

1. **(a)** $x = -2, -1, 5$ **(b)** $-2 < x < -1$ or $x > 5$ **(c)** $x < -2$ or $-1 < x < 5$

3. **(a)** $x = -7, -4, 6$ **(b)** $x < -7$ or $-4 < x < 6$ or $x > 6$ **(c)** $-7 < x < -4$

5. **(a)** $x = 8, -1$ **(b)** $-1 < x < 8$ or $x > 8$ **(c)** $x < -1$ **7.** $(-1, 3) \cup (3, \infty)$ **9.** $(-\infty, -1) \cup (1, 2)$

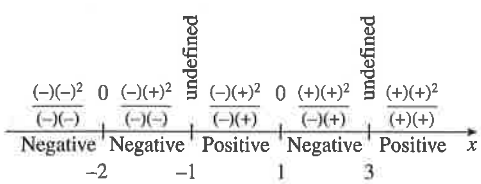
11. $\left[-2, \frac{1}{2}\right] \cup [3, \infty)$ **13.** $[-1, 0] \cup [2, \infty)$ **15.** $\left(-1, \frac{3}{2}\right) \cup (2, \infty)$ **17.** $[-1.15, \infty)$ **19.** $\left(\frac{3}{2}, 2\right)$ **21. (a)** $(-\infty, \infty)$ **(b)** $(-\infty, \infty)$ **(c)** There are no solutions. **(d)** There are no solutions. **23. (a)** $x \neq \frac{4}{3}$ **(b)** $(-\infty, \infty)$ **(c)** There are no solutions. **(d)** $x = \frac{4}{3}$ **25. (a)** $x = 1$ **(b)** $x = -\frac{3}{2}, 4$ **(c)** $-\frac{3}{2} < x < 1$ or $x > 4$ **(d)** $x < -\frac{3}{2}$, or $1 < x < 4$ **27. (a)** $x = 0, -3$ **(b)** $x < -3$ **(c)** $x > 0$ **(d)** $-3 < x < 0$ **29. (a)** $x = -5$ **(b)** $x = -\frac{1}{2}, x = 1, x < -5$ **(c)** $-5 < x < -\frac{1}{2}$ or $x > 1$ **(d)** $-\frac{1}{2} < x < 1$ **31. (a)** $x = 3$ **(b)** $x = 4, x < 3$ **(c)** $3 < x < 4$ or $x > 4$ **(d)** $f(x)$ is never negative **33.** $(-\infty, -2) \cup (1, 2)$ **35.** $[-1, 1]$ **37.** $(-\infty, -4) \cup (3, \infty)$ **39.** $[-1, 0] \cup [1, \infty)$ **41.** $(0, 2) \cup (2, \infty)$ **43.** $\left(-4, \frac{1}{2}\right)$ **45.** $(0, 2)$ **47.** $(-\infty, 0) \cup (\sqrt[3]{2}, \infty)$ **49.** $(-\infty, -1) \cup [1, 3)$ **51.** $[-3, \infty)$ **53.** $[5, \infty)$ **57.** 1 in. $< x <$ 34 in. **59.** 0 in. $\leq x \leq$ 0.69 in. or 4.20 in. $\leq x \leq$ 6 in. **61. (b)** 1.12 cm $\leq x \leq$ 11.37 cm, 1.23 cm $\leq h \leq$ 126.88 cm **(c)** about 348.73 cm^2 **63. (a)** $y \approx 49.103x^2 + 553.447x + 19{,}623.266$ **(b)** 2005



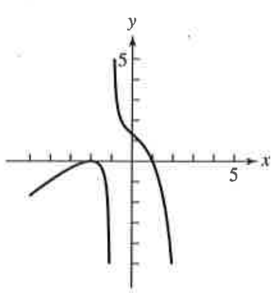
[0, 15] by [0, 50,000]

65. False, because the factor x^4 does not change sign at $x = 0$. **67.** (c) **69.** (d)

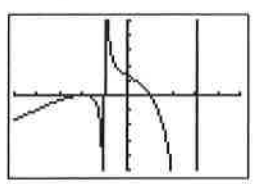
71. Vertical asymptotes: $x = -1, x = 3$; x-intercepts: $(-2, 0)$, $(1, 0)$; y-intercept: $\left(0, \frac{4}{3}\right)$



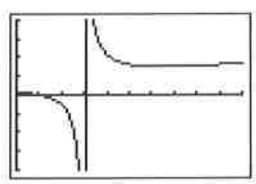
Sketch:



Graph art:



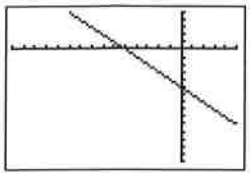
[−5, 5] by [−5, 5]



[0, 10] by [−40, 40]

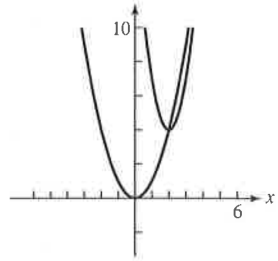
CHAPTER 2 REVIEW EXERCISES

1. $y = -x - 5$



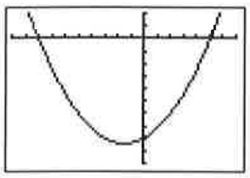
$[-15, 5]$ by $[-15, 5]$

3. Starting from $y = x^2$, translate right 2 units and vertically stretch by 3 (either order), then translate up 4 units.



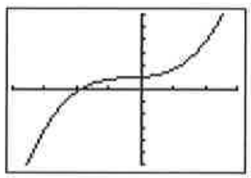
5. Vertex: $(-3, 5)$; axis: $x = -3$ **7.** Vertex: $(-4, 1)$; axis: $x = -4$ **9.** $y = \frac{5}{9}(x + 2)^2 - 3$. **11.** $y = \frac{1}{2}(x - 3)^2 - 2$

13.



$[-10, 7]$ by $[-50, 10]$

15.



$[-4, 3]$ by $[-30, 30]$

17. $S = kr^2$ $(k = 4\pi)$ **19.** The force F needed varies directly with the distance x from its resting position, with constant of variation k. **21.** $k = 4$, $a = \frac{1}{3}$; f is increasing in the first quadrant; f is odd. **23.** $k = -2$, $a = -3$; f is increasing in the fourth quadrant; f is odd. **25.** $2x^2 - x + 1 - \frac{2}{x - 3}$

27. $2x^2 - 3x + 1 + \frac{-2x + 3}{x^2 + 4}$ **29.** -39 **31.** Yes **37.** $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$; $-\frac{3}{2}$ and 2 are zeros. **39.** $1 + 3i$

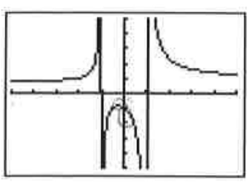
41. $7 + 4i$ **43.** 25 **45.** $4i$ **47.** $3 \pm 2i$ **49.** (c) **51.** (b) **53.** Rational: 0. Irrational: $5 \pm \sqrt{2}$. No nonreal zeros.

55. Rational: none. Irrational: approximately -2.34, 0.57, 3.77. No nonreal zeros. **57.** Rational: $-\frac{3}{2}$. Nonreal complex: $3 + i$. $f(x) = (2x + 3)(x - 3 + i)(x - 3 - i)$. **59.** Rational: $1, -1, \frac{2}{3}$, and $-\frac{5}{2}$. $f(x) = (3x - 2)(2x + 5)(x - 1)(x + 1)$

61. $f(x) = (x - 2)(x^2 + x + 1)$ **63.** $f(x) = (2x - 3)(x - 1)(x^2 - 2x + 5)$ **65.** $x^3 - 3x^2 - 5x + 15$

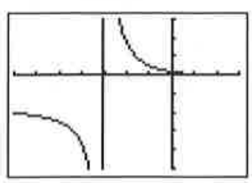
67. $6x^4 - 5x^3 - 38x^2 - 5x + 6$ **69.** $x^4 - 4x^3 - 12x^2 + 32x + 64$ **71.** Translate right 5 units and vertically stretch by 2 (either order), then translate down 1 unit. Horizontal asymptote: $y = -1$; vertical asymptote: $x = 5$.

73. Asymptotes: $y = 1$, $x = -1$, and $x = 1$.
Intercept: $(0, 21)$.



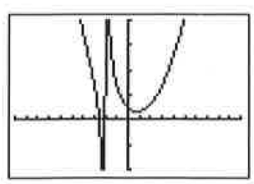
$[-5, 5]$ by $[-5, 5]$

75. End behavior asymptote: $y = x - 7$.
Vertical asymptote: $x = -3$. Intercept: $\left(0, \frac{5}{3}\right)$.



$[-7, 3]$ by $[-50, 30]$

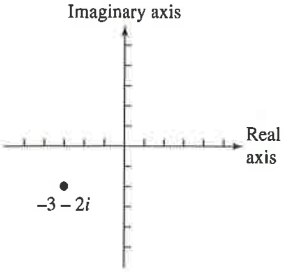
77. y-intercept: $\left(0, \frac{5}{2}\right)$, x-intercept: $(-2.55, 0)$;
Domain: $x \neq -2$; Range: $(-\infty, \infty)$; Continuity: all $x \neq -2$;
Decreasing: $(-\infty, -2)$, $(-2, 0.82)$; Increasing: $(0.82, \infty)$;
Unbounded; Local minimum: $(0.82, 1.63)$;
Vertical asymptote: $x = -2$; End behavior asymptote: $y = x^2 - x$;
$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$



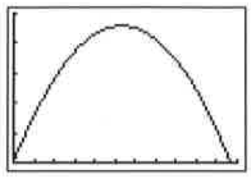
$[-10, 10]$ by $[-10, 20]$

79. $x = \frac{3}{2}, x = 4$ **81.** $\left(-\infty, \frac{5}{2}\right) \cup (-2, 3)$ **83.** $[-3, -2) \cup (2, \infty)$ **85.** $x = -3, x = \frac{1}{2}$

87.



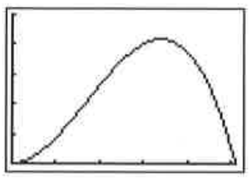
89. (a) $h = -16t^2 + 170t + 6$



[0, 11] by [0, 500]

(b) When $t \approx 5.3125$, $h \approx 457.5625$ **(c)** about 10.66 sec

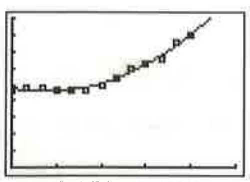
91. (a) & (b)



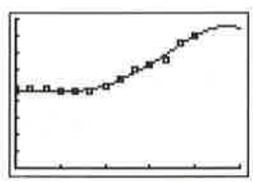
[0, 255] by [0, 2.5]

(c) When $d \approx 170$ ft, $s \approx 2.088$ ft.

93. (a) $y = 18.694x^2 - 88.144x + 2393.0222$ **(b)** $y = -0.291x^4 + 7.100x^3 - 35.865x^2 + 48.971x + 2336.634$



[0, 15] by [0, 4500]



[0, 15] by [0, 4500]

(c) Using quadratic regression: \$5768; Using quartic regression: \$3949

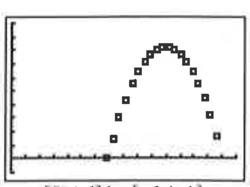
95. (a) $C = 4.32 + \dfrac{4000}{x}$ **(b)** Solve $x\left(5.25 - 4.32 - \dfrac{4000}{x}\right) = 8000$: $0.93x = 12000$, so $x \approx 12\,903.23$ — round up to 12,904.

97. (a) $R_2 = \dfrac{1.2x}{x - 1.2}$ **(b)** 2 ohms **99. (a)** $S = 2\pi x^2 + \dfrac{2000}{x}$ **(b)** Either $x \approx 2.31$ cm and $h \approx 59.75$ cm, or $x \approx 10.65$ and $h \approx 2.81$. **(c)** Approximately $2.31 < x < 10.65$ (graphically) and $2.81 < h < 59.75$.

Chapter 2 Project

Answers are based on the sample data shown in the table.

1.



[0, 1.6] by [−0.1, 1]

3. The sign of a affects the direction the parabola opens.
The magnitude of a affects the vertical stretch of the graph.
Changes to h cause horizontal shifts to the graph while changes to k cause vertical shifts.

5. $y \approx -4.968x^2 + 10.913x - 5.160$