SECTION 4.1

Exploration 1

1. $2\pi r$ **3.** No, not quite, since the distance πr would require a piece of thread π times as long, and $\pi > 3$.

Quick Review 4.1

1. 5π in. **3.** $\frac{6}{\pi}$ m **5.** **(a)** 47.52 ft **(b)** 39.77 km **7.** 88 ft/sec **9.** 6 mph

Exercises 4.1

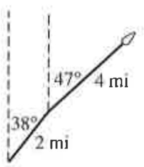
1. 23.2° **3.** 118.7375° **5.** 21°12′ **7.** 118°19′12″ **9.** $\frac{\pi}{3}$ rad **11.** $\frac{2\pi}{3}$ rad **13.** ≈ 1.2518 rad **15.** ≈ 1.0716 rad

17. 30° **19.** 18° **21.** 140° **23.** $\approx$ 114.59° **25.** 50 in. **27.** $\frac{6}{\pi}$ ft **29.** 3 radians **31.** $\frac{360}{\pi}$ cm

33. $\theta = \frac{9}{11}$ rad and $s_2 = 36$ cm **35.** 24 in. **37.** $\approx$ 5.4 in. **39.** **(a)** 45° **(b)** 22.5° **(c)** 247.5°

41. ESE is closest at 112.5°. **43.** $\approx$4.23 statute miles **45.** $\approx$387.85 rpm **47.** $\approx$12,566.37

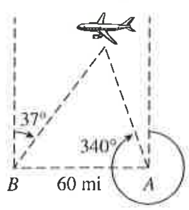
49.



51. $\approx$ 778 nautical miles **53.** **(a)** $16\pi \approx 50.265$ in. **(b)** $2\pi \approx 6.283$ ft
55. **(a)** 4π rad/sec **(b)** 28π cm/sec **(c)** 7π rad/sec
57. True. Horse A travels $2\pi(2r) = 2(2\pi r)$ units of distance in the same amount of time that horse B travels $2\pi r$ units of distance, and so is moving twice as fast. **59.** (c) **61.** (b) **63.** 38°02′
65. 5°37′ **67.** 80 naut mi **69.** 902 naut mi

71. The whole circle's area is πr^2; the sector with central angle θ makes up $\frac{\theta}{2\pi}$ of that area, or $\frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}\theta r^2$.

73.



SECTION 4.2

Exploration 1

1. sin and csc, cos and sec, and tan and cot **3.** sec θ **5.** sin θ and cos θ

Exploration 2

1. Let $\theta = 60°$. Then,

$$\sin\theta = \frac{\sqrt{3}}{2} \approx 0.866 \qquad \csc\theta = \frac{2}{\sqrt{3}} \approx 1.155$$

$$\cos\theta = \frac{1}{2} \qquad \sec\theta = 2$$

$$\tan\theta = \sqrt{3} \approx 1.732 \qquad \cot\theta = \frac{1}{\sqrt{3}} \approx 0.577$$

3. The value of a trig function at θ is the same as the value of its co-function at $90° - \theta$.

Quick Review 4.2

1. $5\sqrt{2}$ **3.** 6 **5.** 100.8 in. **7.** 7.9152 km **9.** $\approx$ 1.0101 (no units)

Exercises 4.2

1. $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = \frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = \frac{3}{4}$ **3.** $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}, \csc\theta = \frac{13}{12},$ $\sec\theta = \frac{13}{5}, \cot\theta = \frac{5}{12}$ **5.** $\sin\theta = \frac{7}{\sqrt{170}}, \cos\theta = \frac{11}{\sqrt{170}}, \tan\theta = \frac{7}{11}, \csc\theta = \frac{\sqrt{170}}{7}, \sec\theta = \frac{\sqrt{170}}{11}, \cot\theta = \frac{11}{7}$ **7.** $\sin\theta = \frac{\sqrt{57}}{11}, \cos\theta = \frac{8}{11}, \tan\theta = \frac{\sqrt{57}}{8}, \csc\theta = \frac{11}{\sqrt{57}}, \sec\theta = \frac{11}{8}, \cot\theta = \frac{8}{\sqrt{57}}$ **9.** $\cos\theta = \frac{2\sqrt{10}}{7}, \tan\theta = \frac{3}{2\sqrt{10}},$ $\csc\theta = \frac{7}{3}, \sec\theta = \frac{7}{2\sqrt{10}}, \cot\theta = \frac{2\sqrt{10}}{3}$ **11.** $\sin\theta = \frac{4\sqrt{6}}{11}, \tan\theta = \frac{4\sqrt{6}}{5}, \csc\theta = \frac{11}{4\sqrt{6}}, \sec\theta = \frac{11}{5}, \cot\theta = \frac{5}{4\sqrt{6}}$ **13.** $\sin\theta = \frac{5}{\sqrt{106}}, \cos\theta = \frac{9}{\sqrt{106}}, \csc\theta = \frac{\sqrt{106}}{5}, \sec\theta = \frac{\sqrt{106}}{9}, \cot\theta = \frac{9}{5}$ **15.** $\sin\theta = \frac{3}{\sqrt{130}}, \cos\theta = \frac{11}{\sqrt{130}}, \tan\theta = \frac{3}{11},$ $\csc\theta = \frac{\sqrt{130}}{3}, \sec\theta = \frac{\sqrt{130}}{11}$ **17.** $\sin\theta = \frac{9}{23}, \cos\theta = \frac{8\sqrt{7}}{23}, \tan\theta = \frac{9}{8\sqrt{7}}, \sec\theta = \frac{23}{8\sqrt{7}}, \cot\theta = \frac{8\sqrt{7}}{9}$ **19.** $\frac{\sqrt{3}}{2}$ **21.** $\sqrt{3}$ **23.** $\frac{\sqrt{2}}{2}$ **25.** $\sqrt{2}$ **27.** $\approx \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ **29.** ≈ 0.961 **31.** ≈ 0.943 **33.** ≈ 0.268 **35.** ≈ 1.524 **37.** ≈ 0.810 **39.** ≈ 2.414 **41.** $30° = \frac{\pi}{6}$ **43.** $60° = \frac{\pi}{3}$ **45.** $60° = \frac{\pi}{3}$ **47.** $30° = \frac{\pi}{6}$ **49.** $\frac{15}{\sin 34°} \approx 26.82$ **51.** $\frac{32}{\tan 57°} \approx 20.78$ **53.** $\frac{6}{\sin 35°} \approx 10.46$ **55.** $b \approx 33.79, c \approx 35.96, \beta = 70°$ **57.** $b \approx 22.25, c \approx 27.16, \alpha = 35°$ **59.** As θ gets smaller and smaller, the side opposite θ gets smaller and smaller, so its ratio to the hypotenuse approaches 0 as a limit. **61.** ≈ 205.26 ft **63.** $\approx 74.16\ \text{ft}^2$ **65.** ≈ 378.80 ft **67.** False. This is only true if θ is an acute angle in a right triangle. **69.** (e) **71.** (d) **73.** ≈ 2.69 m

75. One possible proof:

$$\begin{aligned}(\sin\theta)^2 + (\cos\theta)^2 &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\ &= \frac{a^2}{c^2} + \frac{b^2}{c^2} \\ &= \frac{a^2 + b^2}{c^2} \\ &= \frac{c^2}{c^2} \quad \text{(Pythagorean theorem: } a^2 + b^2 = c^2.\text{)} \\ &= 1\end{aligned}$$

SECTION 4.3

Exploration 1

1. The side opposite θ in the triangle has length y and the hypotenuse has length r. Therefore, $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$. **3.** $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$

Exploration 2

1. The x-coordinates on the unit circle lie between -1 and 1, and $\cos t$ is always an x-coordinate on the unit circle. **3.** The points corresponding to t and $-t$ on the number line are wrapped to points above and below the x-axis with the same x-coordinates. Therefore, $\cos t$ and $\cos(-t)$ are equal. **5.** Since 2π is the distance around the unit circle, both t and $t + 2\pi$ get wrapped to the same point. **7.** By the observation in (6), $\tan t$ and $\tan(t + \pi)$ are ratios of the form $\frac{y}{x}$ and $\frac{-y}{-x}$, which are either equal to each other or both undefined. **9.** Answers will vary. For example, there are similar statements that can be made about the functions cot, sec, and csc.

Quick Review 4.3

1. $-30°$ **3.** $45°$ **5.** $\frac{\sqrt{3}}{3}$ **7.** $\sqrt{2}$ **9.** $\cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}, \csc\theta = \frac{13}{5}, \sec\theta = \frac{13}{12}, \cot\theta = \frac{12}{5}$

Exercises 4.3

1. 450° **3.** $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = -2$, $\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = -\frac{1}{2}$

5. $\sin\theta = -\frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$, $\tan\theta = 1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = -\sqrt{2}$, $\cot\theta = 1$

7. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

9. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta$ undefined, $\csc\theta = 1$, $\sec\theta$ undefined, $\cot\theta = 0$

11. $\sin\theta = -\frac{2}{\sqrt{29}}$, $\cos\theta = \frac{5}{\sqrt{29}}$, $\tan\theta = -\frac{2}{5}$, $\csc\theta = -\frac{\sqrt{29}}{2}$, $\sec\theta = \frac{\sqrt{29}}{5}$, $\cot\theta = -\frac{5}{2}$

13. **(a)** + **(b)** + **(c)** + **15.** **(a)** − **(b)** − **(c)** + **17.** − **19.** − **21.** (a) **23.** (a) **25.** $-\frac{1}{2}$ **27.** 2

29. $\frac{1}{2}$ **31.** 1 **33.** $\frac{\sqrt{3}}{2}$ **35.** $-\frac{\sqrt{3}}{2}$ **37.** **(a)** −1 **(b)** 0 **(c)** undefined **39.** **(a)** 0 **(b)** −1 **(c)** 0

41. **(a)** 1 **(b)** 0 **(c)** undefined **43.** $\sin\theta = \frac{\sqrt{5}}{3}$ and $\tan\theta = \frac{\sqrt{5}}{2}$ **45.** $\tan\theta = -\frac{2}{\sqrt{21}}$ and $\sec\theta = \frac{5}{\sqrt{21}}$

47. $\sec\theta = -\frac{5}{4}$ and $\csc\theta = \frac{5}{3}$ **49.** $\frac{1}{2}$ **51.** 0 **53.** The calculator's value of the irrational number π is necessarily an approximation. When multiplied by a very large number, the slight error of the original approximation is magnified sufficiently to throw the trigonometric functions off. **55.** $\mu = \frac{\sin 83°}{\sin 36°} \approx 1.69$ **57.** **(a)** 0.4 in. **(b)** ≈ 0.1852 in.

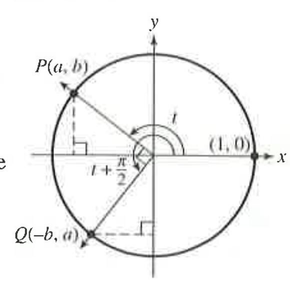
59. The difference in the elevations is 600 ft, so $d = 600/\sin\theta$. Then: **(a)** ≈ 848.53 ft **(b)** 600 ft **(c)** ≈ 933.43 ft

61. True. Acute angles determine reference triangles in QI, where cosine is positive, while obtuse angles determine reference triangles in QII, where cosine is negative. **63.** (e) **65.** (a) **67.** $\frac{5\pi}{6}$ **69.** $\frac{7\pi}{4}$

71. The two triangles are congruent: both have hypotenuse 1, and the corresponding angles are congruent—the smaller acute angle has measure t in both triangles, and the two acute angles in a right triangle add up to $\pi/2$.

73. One possible answer: Starting from the point (a, b) on the unit circle—at an angle of t, so that $\cos t = a$—then measuring a quarter of the way around the circle (which corresponds to adding $\pi/2$ to the angle), we end at $(-b, a)$, so that $\sin\left(t + \frac{\pi}{2}\right) = a$. For (a, b) in quadrant I, this is shown in the figure; similar illustrations can be drawn for the other quadrants.

75. Starting from the point (a, b) on the unit circle — at an angle of t, so that $\cos t = a$—then measuring a quarter of the way around the circle (which corresponds to adding $\pi/2$ to the angle), we end at $(-b, a)$, so that $\sin(t + \pi/2) = a$. This holds true when (a, b) is in quadrant II, just as it did for quadrant I.



77. $|\theta| < 0.2441$ (approximately)

79. This Taylor polynomial is generally a very good approximation for $\sin\theta$—in fact, the relative error is less than 1% for $|\theta| < 1$ (approx.). It is better for θ close to 0; it is slightly larger than $\sin\theta$ when $\theta < 0$ and slightly smaller when $\theta > 0$.

SECTION 4.4

Exploration 1

1. $\pi/2$ (at the point (0, 1)) **3.** Both graphs cross the x-axis when the y-coordinate on the unit circle is 0.

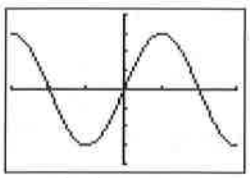
5. The sine function tracks the y-coordinate of the point as it moves around the unit circle. After the point has gone completely around the unit circle (a distance of 2π), the same pattern of y-coordinates starts over again.

Quick Review 4.4

1. In order: +, +, −, − **3.** In order: +, −, +, − **5.** $-\frac{5\pi}{6}$ **7.** Vertically stretch by 3. **9.** Vertically shrink by 0.5.

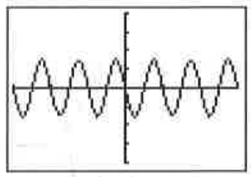
Exercises 4.4

1. Amplitude 2; vertical stretch by a factor of 2 **3.** Amplitude 4; vertical stretch by a factor of 4, reflection across x-axis **5.** Amplitude 0.73; vertical shrink by a factor of 0.73 **7.** Period $\frac{2\pi}{3}$; horizontal shrink by a factor of $\frac{1}{3}$ **9.** Period $\frac{2\pi}{7}$; horizontal shrink by a factor of $\frac{1}{7}$, reflection across y-axis **11.** Period π; horizontal shrink by a factor of $\frac{1}{2}$, vertical stretch by a factor of 3 **13.** Amplitude 3, period 4π, frequency $\frac{1}{4\pi}$



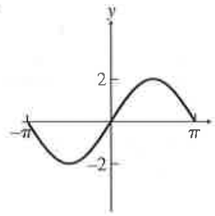
[-3π, 3π] by [-4, 4]

15. Amplitude $\frac{3}{2}$, period π, frequency $\frac{1}{\pi}$

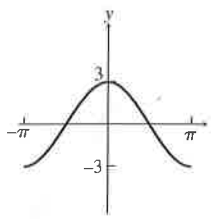


[-3π, 3π] by [-4, 4]

17.



19.



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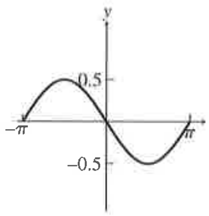

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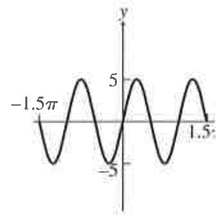

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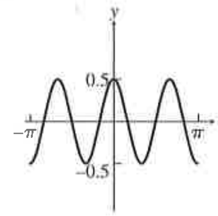
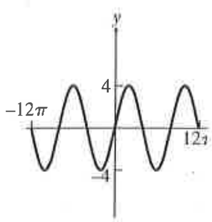
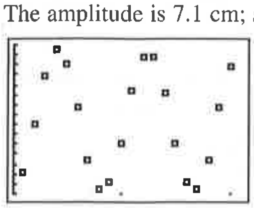


27.



29. Period π; amplitude 1.5; $[-2\pi, 2\pi]$ by $[-2, 2]$ **31.** Period π; amplitude 3; $[-2\pi, 2\pi]$ by $[-4, 4]$ **33.** Period 6; amplitude 4; $[-3, 3]$ by $[-5, 5]$ **35.** Maximum: 2 $\left(\text{at } -\frac{3\pi}{2} \text{ and } \frac{\pi}{2}\right)$; minimum: -2 $\left(\text{at } -\frac{\pi}{2} \text{ and } \frac{3\pi}{2}\right)$; zeros: $0, \pm\pi, \pm 2\pi$ **37.** Maximum: 1 (at $0, \pm\pi, \pm 2\pi$); minimum: -1 $\left(\text{at } \pm\frac{\pi}{2} \text{ and } \pm\frac{3\pi}{2}\right)$; zeros: $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$ **39.** Maximum: 1 $\left(\text{at } \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right)$; minimum: -1 (at $0, \pm\pi, \pm 2\pi$); zeros: $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$ **41.** One possibility is $y = \sin(x + \pi)$. **43.** Starting from $y = \sin x$, horizontally shrink by $\frac{1}{3}$ and vertically shrink by 0.5. **45.** Starting from $y = \cos x$, horizontally stretch by 3, vertically shrink by $\frac{2}{3}$, reflect across x-axis. **47.** Starting from $y = \cos x$, horizontally shrink by $\frac{3}{2\pi}$ and vertically stretch by 3. **49.** Starting with y_1, vertically stretch by $\frac{5}{3}$. **51.** Starting with y_1, horizontally shrink by $\frac{1}{2}$. **53.** (a) and (b) **55.** (a) and (b) **57.** One possibility is $y = 3 \sin 2x$. **59.** One possibility is $y = 1.5 \sin 12(x - 1)$. **61.** Amplitude 2, period 2π, phase shift $\frac{\pi}{4}$, vertical translation 1 unit up **63.** Amplitude 5, period $\frac{2\pi}{3}$, phase shift $\frac{\pi}{18}$, vertical translation $\frac{1}{2}$ units up **65.** Amplitude 2, period 1, phase shift 0, vertical translation 1 unit up **67.** Amplitude $\frac{7}{3}$, period 2π, phase shift $-\frac{5}{2}$, vertical translation 1 unit down **69.** $y = 2 \sin 2x$ ($a = 2, b = 2, h = 0, k = 0$) **71.** **(a)** two **(b)** (0, 1) and $(2\pi, 1.3^{-2\pi}) \approx (6.28, 0.19)$. **73.** ≈ 15.90 sec. **75.** **(a)** 1:00 A.M. **(b)** 8.90 ft; 10.52 ft **(c)** 4:06 A.M.

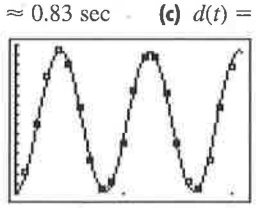
77. **(a)** The maximum d is approximately 21.4 cm. The amplitude is 7.1 cm; scatterplot:



[0, 2.1] by [7, 22]

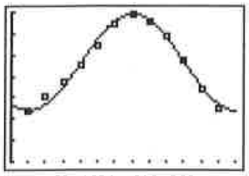
(b) ≈ 0.83 sec **(c)** $d(t) = -7.1 \cos \frac{2\pi x}{0.83} + 14.3$

(d)



[0, 2.1] by [7, 22]

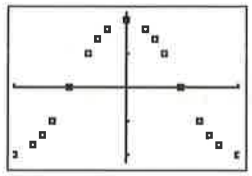
79. One possible solution is $T = 22.5\cos\left(\frac{\pi}{6}(x - 7)\right) + 56.5.$



[0, 13] by [10, 80]

81. False. Since $y = \sin 2x$ is a horizontal stretch of $y = \sin 4x$ by a factor of 2, it has twice the period. **83.** (d) **85.** (c)

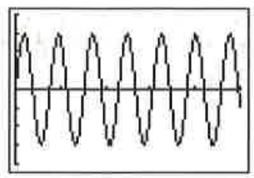
87. **(a)**



$[-\pi, \pi]$ by $[-1.1, 1.1]$

(b) $0.0246x^4 + 0x^3 - 0.4410x^2 + 0x + 0.9703$
(c) The coefficients are fairly similar.

89. **(a)** $\frac{1}{262}$ sec **(b)** $f = 262\,\frac{1}{\text{sec}}$ ("cycles per sec"), or 262 Hertz **(c)**



[0, 0.025] by [−2, 2]

91. **(a)** $a - b$ must equal 1. **(b)** $a - b$ must equal 2. **(c)** $a - b$ must equal k.

93. $B = (0, 3)$, $C = \left(\frac{3\pi}{4}, 0\right)$ **95.** $B = \left(\frac{\pi}{4}, 2\right)$, $C = \left(\frac{3\pi}{4}, 0\right)$

SECTION 4.5

Exploration 1

1. The graphs do not seem to intersect.

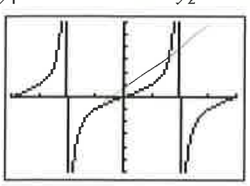
Quick Review 4.5

1. π **3.** 6π **5.** Zero; 3; asymptote: $x = -4$ **7.** Zero; -1; asymptotes: $x = 2$ and $x = -2$ **9.** Even

Exercises 4.5

1. The graph of $y = 2\csc x$ must be vertically stretched by 2 compared to $y = \csc x$, so $y_1 = 2\csc x$ and $y_2 = \csc x$.
3. $y_1 = 3\csc 2x$ and $y_2 = \csc x$

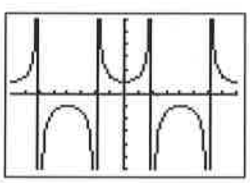
5.



$[-\frac{\pi}{2}, \frac{\pi}{2}]$ by $[-6, 6]$

Horizontal shrink of $y = \tan x$ by factor 1/2; asymptotes at odd multiples of $\pi/4$

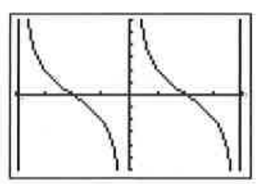
7.



$[-\frac{2\pi}{3}, \frac{2\pi}{3}]$ by $[-6, 6]$

Horizontal shrink of $y = \sec x$ by factor 1/3; asymptotes at odd multiples of $\pi/6$

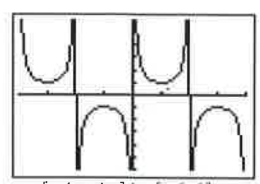
9.



$[-\frac{\pi}{2}, \frac{\pi}{2}]$ by $[-6, 6]$

Horizontal shrink of $y = \cot x$ by factor 1/2, vertical stretch by factor 2; asymptotes at multiples of $\pi/2$

11.



$[-4\pi, 4\pi]$ by $[-6, 6]$

Horizontal stretch of $y = \csc x$ by factor 2; asymptotes at multiples of 2π

13. Domain: All reals except integer multiples of π; Range: $(-\infty, \infty)$; Continuous on its domain; Decreasing on each interval in its domain; Symmetry with respect to the origin (odd); Not bounded above or below; No local extrema; No horizontal asymptotes; Vertical asymptotes: $x = k\pi$ for all integers k; End behavior: $\lim_{x\to\infty} \cot x$ and $\lim_{x\to-\infty} \cot x$ do not exist.

15. Domain: All reals except integer multiples of π; Range: $(-\infty, -1] \cup [1, \infty)$; Continuous on its domain; On each interval centered at $x = \frac{\pi}{2} + 2k\pi$, where k is an integer, decreasing on the left half of the interval and increasing on the right, for $x = \frac{3\pi}{2} + 2k\pi$, increasing on the first half of the interval and decreasing on the second half; Symmetric with respect to the origin (odd); Not bounded above or below; Local minimum 1 at each $x = \frac{\pi}{2} + 2k\pi$ and local maximum -1 at each $x = \frac{3\pi}{2} + 2k\pi$, where k is an integer; No horizontal asymptotes; Vertical asymptotes: $x = k\pi$ for all integers k; End behavior: $\lim_{x\to\infty} \csc x$ and $\lim_{x\to-\infty} \csc x$ do not exist.

17. Graph (a); Xmin $= -\pi$ and Xmax $= \pi$ **19.** Graph (c); Xmin $= -\pi$ and Xmax $= \pi$ **21.** Starting with $y = \tan x$, vertically stretch by 3. **23.** Starting with $y = \csc x$, vertically stretch by 3. **25.** Starting with $y = \cot x$, horizontally stretch by 2, vertically stretch by 3, and reflect across x-axis. **27.** Starting with $y = \tan x$, horizontally shrink by $\frac{2}{\pi}$, reflect across x-axis, and shift up by 2 units. **29.** $\frac{\pi}{3}$ **31.** $\frac{5\pi}{6}$ **33.** $\frac{5\pi}{2}$ **35.** $x \approx 0.92$ **37.** $x \approx 5.25$ **39.** $x \approx 0.52$ or $x \approx 2.62$

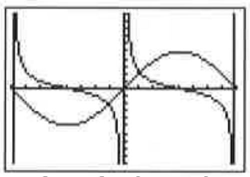
41. **(a)** The reflection of (a, b) across the origin is $(-a, -b)$. **(b)** Definition of tangent **(c)** $\tan t = \frac{b}{a} = \frac{-b}{-a} = \tan(t - \pi)$

(d) Since points on opposite sides of the unit circle determine the same tangent ratio, $\tan(t \pm \pi) = \tan t$ for all numbers t in the domain. Other points on the unit circle yield triangles with different tangent ratios, so no smaller period is possible.

(e) The same argument that uses the ratio $\frac{b}{a}$ above can be repeated using the ratio $\frac{a}{b}$, which is the cotangent ratio.

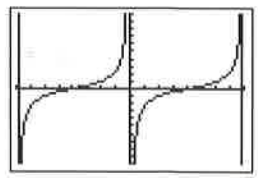
43. For any x, $\left(\frac{1}{f}\right)(x + p) = \frac{1}{f(x + p)} = \frac{1}{f(x)} = \left(\frac{1}{f}\right)(x)$. This is not true for any smaller value of p, since this is the smallest value that works for f. **45.** **(a)** $d = 350 \sec x$ **(b)** $\approx 16{,}831$ ft. **47.** $\approx \pm 0.905$ **49.** $\approx \pm 1.107$ or $\approx \pm 2.034$ **51.** False. It is increasing only over intervals on which it is defined, i.e., intervals bounded by consecutive asymptotes. **53.** (a) **55.** (d)

57. About $(-0.44, 0) \cup (0.44, \pi)$



$[-\pi, \pi]$ by $[-10, 10]$

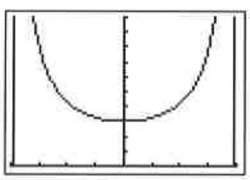
59. $\cot x$ is not defined at 0; the definition of "increasing on (a, b)" requires that the function be defined everywhere in (a, b). Also, choosing $a = -\pi/4$ and $b = \pi/4$, we have $a < b$ but $f(a) = 1 > f(b) = -1$.



$[-\pi, \pi]$ by $[-10, 10]$

61. $\csc x = \sec\left(x - \frac{\pi}{2}\right)$ (or $\csc x = \sec\left(x - \left(\frac{\pi}{2} + n\pi\right)\right)$ for any integer n) This is a translation to the right of $\frac{\pi}{2}$ $\left(\text{or } \frac{\pi}{2} + n\pi\right)$ units.

63. $d = 30 \sec x = \frac{30}{\cos x}$

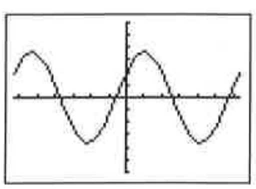


$[-.5\pi, .5\pi]$ by $[0, 100]$

65. ≈ 0.8952 radians $\approx 51.29°$

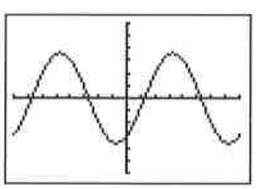
SECTION 4.6

Exploration 1



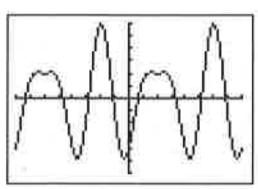
$[-2\pi, 2\pi]$ by $[-6, 6]$

Sinusoid



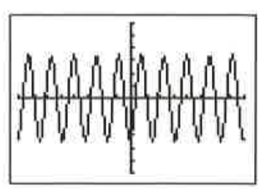
$[-2\pi, 2\pi]$ by $[-6, 6]$

Sinusoid



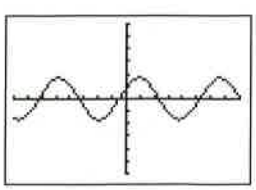
$[-2\pi, 2\pi]$ by $[-6, 6]$

Not a Sinusoid



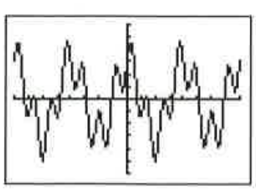
$[-2\pi, 2\pi]$ by $[-6, 6]$

Sinusoid



$[-2\pi, 2\pi]$ by $[-6, 6]$

Sinusoid



$[-2\pi, 2\pi]$ by $[-6, 6]$

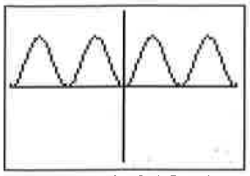
Not a Sinusoid

Quick Review 4.6

1. Domain: $(-\infty, \infty)$; range: $[-3, 3]$ **3.** Domain: $[1, \infty)$; range: $[0, \infty)$ **5.** Domain: $(-\infty, \infty)$; range: $[-2, \infty)$
7. As $x \to -\infty, f(x) \to \infty$; as $x \to \infty, f(x) \to 0$.
9. $(f \circ g)(x) = x - 4$, domain: $[0, \infty)$; $(g \circ f)(x) = \sqrt{x^2 - 4}$, domain: $(-\infty, -2] \cup [2, \infty)$

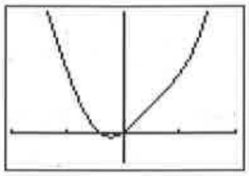
Exercises 4.6

1. Periodic



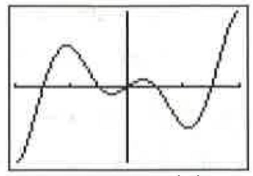
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

3. Not periodic



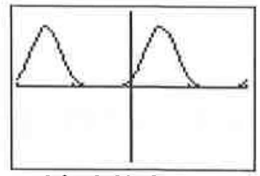
$[-2\pi, 2\pi]$ by $[-5, 20]$

5. Not periodic



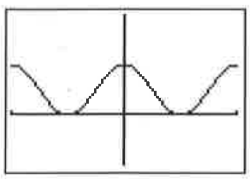
$[-2\pi, 2\pi]$ by $[-6, 6]$

7. Periodic



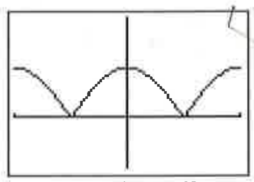
$[-2\pi, 2\pi]$ by $[-10, 10]$

9. Since the period of $\cos x$ is 2π, we have $\cos^2(x + 2\pi) = (\cos(x + 2\pi))^2 = (\cos x)^2 = \cos^2 x$. The period is therefore an exact divisor of 2π, and we see graphically that it is π. A graph for $-\pi \le x \le \pi$ is shown:



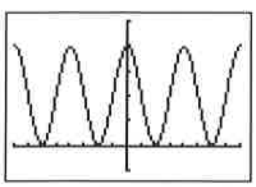
$[-\pi, \pi]$ by $[-1, 2]$

11. Since the period of $\cos x$ is 2π, we have $\sqrt{\cos^2(x + 2\pi)} = \sqrt{(\cos(x + 2\pi))^2} = \sqrt{(\cos x)^2} = \sqrt{\cos^2 x}$. The period is therefore an exact divisor of 2π, and we see graphically that it is π. A graph for $-\pi \le x \le \pi$ is shown:



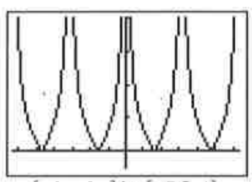
$[-\pi, \pi]$ by $[-1, 2]$

13. Domain: $(-\infty, \infty)$. Range: $[0, 1]$.



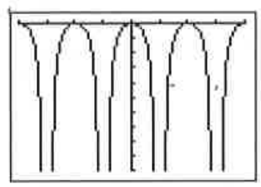
$[-2\pi, 2\pi]$ by $[-0.25, 1.25]$

15. Domain: all $x \ne n\pi$, n an integer. Range: $[0, \infty)$.



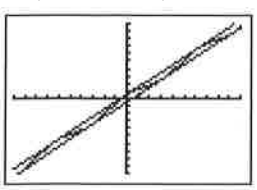
$[-2\pi, 2\pi]$ by $[-0.5, 4]$

17. Domain: all $x \ne \frac{\pi}{2} + n\pi$, n an integer. Range: $(-\infty, 0]$.



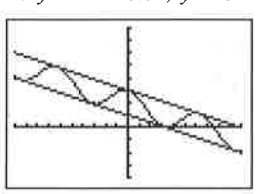
$[-2\pi, 2\pi]$ by $[-10, 0.2]$

19. $y = 2x - 1$; $y = 2x + 1$



$[-10, 10]$ by $[-20, 20]$

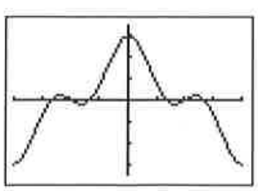
21. $y = 1 - 0.3x$; $y = 3 - 0.3x$



$[-10, 10]$ by $[-4, 8]$

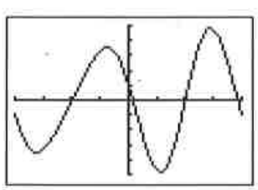
23. Yes **25.** Yes **27.** No
29. $a \approx 3.61, b = 2, h \approx 0.49$
31. $a \approx 2.24, b = \pi, h \approx 0.35$
33. $a \approx 2.24, b = 1, h \approx -1.11$

35.



$[-\pi, \pi]$ by $[-3.5, 3.5]$

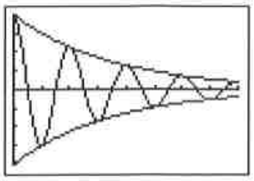
37.



$[-\pi, \pi]$ by $[-5, 5]$

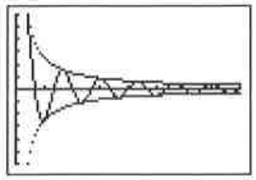
39. (a) **41.** (c)
43. The damping factor is e^{-x}, and the damping occurs as $x \to \infty$.
45. No damping.
47. The damping factor is x^3, and the damping occurs as $x \to 0$.

49. f oscillates between 1.2^{-x} and -1.2^{-x}. As $x \to \infty, f(x) \to 0$.



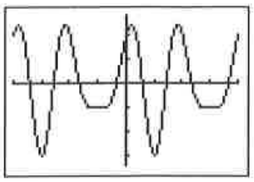
$[0, 4\pi]$ by $[-1, 1]$

51. f oscillates between $\frac{1}{x}$ and $-\frac{1}{x}$. As $x \to \infty, f(x) \to 0$.



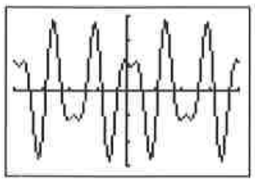
$[0, 4\pi]$ by $[-1.5, 1.5]$

53. 2π



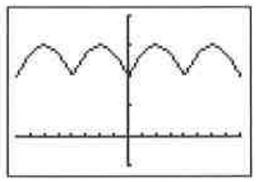
$[-2\pi, 2\pi]$ by $[-3.4, 2.8]$

55. 2π



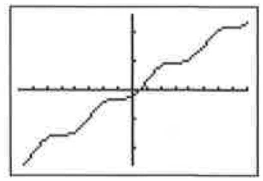
$[-2\pi, 2\pi]$ by $[-3, 3]$

57. Period 2π



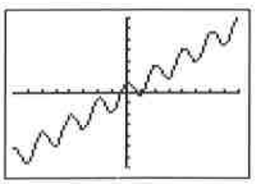
$[-4\pi, 4\pi]$ by $[-1, 4]$

59. Not periodic.



$[-4\pi, 4\pi]$ by $[-13, 13]$

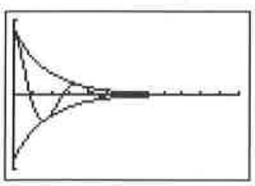
61. Not periodic.



$[-4\pi, 4\pi]$ by $[-7, 7]$

63. Domain is $(-\infty, \infty)$; Range: $(-\infty, \infty)$
65. Domain is $(-\infty, \infty)$; Range: $[1, \infty)$
67. Domain is $\ldots \cup [-2\pi, -\pi] \cup [0, \pi] \cup [2\pi, 3\pi] \cup \ldots$; that is, all x with $2n\pi \le x \le (2n + 1)\pi$, n an integer; Range: $[0, 1]$
69. Domain is $(-\infty, \infty)$; Range: $[0, 1]$

71. (a)



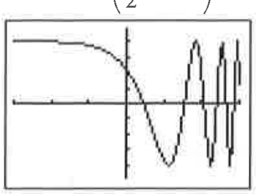
$[0, 12]$ by $[-0.5, 0.5]$

(b) For $t > 0.51$ (approximately)

73. Not periodic **75.** (a)
77. Graph (d), shown on $[-2\pi, 2\pi]$ by $[-4, 4]$
79. Graph (b), shown on $[-2\pi, 2\pi]$ by $[-4, 4]$
81. False. For example, the function has a relative minimum of 0 at $x = 0$ that is not repeated anywhere else. **83.** (b) **85.** (d)

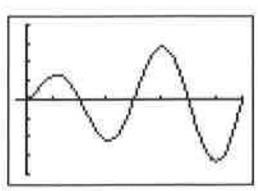
87. (a) Answers will vary — for example, on a TI-81: $\frac{\pi}{47.5} = 0.0661\ldots \approx 0.07$; on a TI-82: $\frac{\pi}{47} = 0.0668\ldots \approx 0.07$; on a TI-85: $\frac{\pi}{63} = 0.0498\ldots \approx 0.05$; on a TI-92: $\frac{\pi}{119} = 0.0263\ldots \approx 0.03$ **(b)** Period: $p = \pi/125 = 0.0251\ldots$. For any of the TI graphers, there are from 1 to 3 cycles between each pair of pixels; the graphs produced are therefore inaccurate, since so much detail is lost.

89. Domain: $(-\infty, \infty)$. Range: $[-1, 1]$. Horizontal asymptote: $y = 1$. Zeros at $\ln\left(\frac{\pi}{2} + n\pi\right)$, n a non-negative integer.



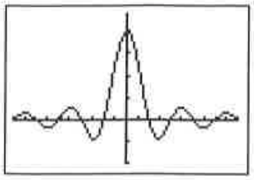
$[-3, 3]$ by $[-1.2, 1.2]$

91. Domain: $[0, \infty)$. Range: $(-\infty, \infty)$. Zeros at $n\pi$, n a nonnegative integer.



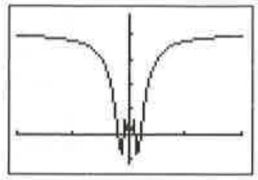
$[-0.5, 4\pi]$ by $[-4, 4]$

93. Domain: $(-\infty, 0) \cup (0, \infty)$. Range: approximately $[-0.22, 1)$. Horizontal asymptote: $y = 0$. Zeros at $n\pi$, n a nonzero integer.



$[-5\pi, 5\pi]$ by $[-0.5, 1.2]$

95. Domain: $(-\infty, 0) \cup (0, \infty)$. Range: approximately $[-0.22, 1)$. Horizontal asymptote: $y = 1$. Zeros at $\frac{1}{n\pi}$, n a nonzero integer.



$[-\pi, \pi]$ by $[-0.3, 1.2]$

SECTION 4.7

Exploration 1

1. x **3.** $\sqrt{1 + x^2}$ **5.** $\sqrt{1 + x^2}$

Quick Review 4.7

1. $\sin x$: positive; $\cos x$: positive; $\tan x$: positive **3.** $\sin x$: negative; $\cos x$: negative; $\tan x$: positive **5.** $\frac{1}{2}$ **7.** $-\frac{1}{2}$ **9.** $-\frac{1}{2}$

Exercises 4.7

1. $\frac{\pi}{3}$ **3.** 0 **5.** $\frac{\pi}{3}$ **7.** $-\frac{\pi}{4}$ **9.** $-\frac{\pi}{4}$ **11.** $\frac{\pi}{2}$ **13.** $\approx 21.22°$ **15.** $\approx -85.43°$ **17.** ≈ 1.172 **19.** ≈ -0.478

21. $\lim_{x \to \infty} \tan^{-1}(x^2) = \frac{\pi}{2}$ and $\lim_{x \to -\infty} \tan^{-1}(x^2) = \frac{\pi}{2}$ **23.** $\frac{\sqrt{3}}{2}$ **25.** $\frac{\pi}{4}$ **27.** $\frac{1}{2}$ **29.** $\frac{\pi}{6}$ **31.** $\frac{1}{2}$

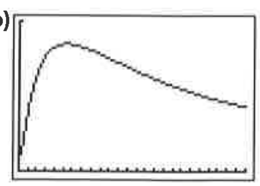
33. Domain: $[-1, 1]$; Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; Continuous; Increasing; Symmetric with respect to the origin (odd); Bounded; Absolute maximum of $\frac{\pi}{2}$, absolute minimum of $-\frac{\pi}{2}$; No asymptotes; No end behavior (bounded domain)

35. Domain: $(-\infty, \infty)$; Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; Continuous; Increasing; Symmetric with respect to the origin (odd); Bounded; No local extrema; Horizontal asymptotes: $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$; End behavior: $\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$ and $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

37. Domain: $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Starting from $y = \sin^{-1} x$, horizontally shrink by $\frac{1}{2}$. **39.** Domain: $(-\infty, \infty)$. Range: $\left(-\frac{5\pi}{2}, \frac{5\pi}{2}\right)$. Starting from $y = \tan^{-1} x$, horizontally stretch by 2 and vertically stretch by 5 (either order). **41.** 1

43. $\sin \frac{1}{2} \approx 0.479$ **45.** $\frac{1}{3}$ **47.** $\frac{x}{\sqrt{1 + x^2}}$ **49.** $\frac{x}{\sqrt{1 - x^2}}$ **51.** $\frac{1}{\sqrt{1 + 4x^2}}$

53. (b)



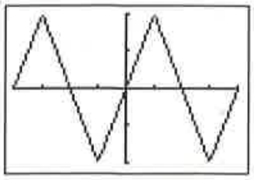
$[0, 25]$ by $[0, 55]$

(c) 2 or 15 ft **55. (a)** $\theta = \tan^{-1} \frac{s}{500}$ **(b)** As s changes from 10 to 20 ft, θ changes from about 1.1458° to 2.2906°—it almost exactly doubles (a 99.92% increase). As s changes from 200 to 210 ft, θ changes from about 21.80° to 22.78°—an increase of less than 1°, and a very small relative change (only about 4.5%). **(c)** The x-axis represents the height and the y-axis represents the angle: the angle cannot grow past 90° (in fact, it *approaches* but never exactly equals 90°).

57. False. This is only true for $-1 \leq x \leq 1$, the domain of the $\sin^{-1}x$ function. **59.** (e) **61.** (c)

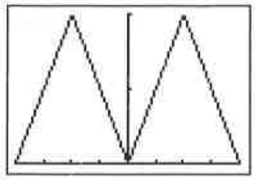
63. The cotangent function restricted to the interval $(0, \pi)$ is one-to-one and has an inverse. The unique angle y between 0 and π (non-inclusive) such that $\cot y = x$ is called the inverse cotangent (or arccotangent) of x, denoted $\cot^{-1} x$ or arccot x. The domain of $y = \cot^{-1} x$ is $(-\infty, \infty)$ and the range is $(0, \pi)$.

65. **(a)** Domain all reals, range $[-\pi/2, \pi/2]$, period 2π



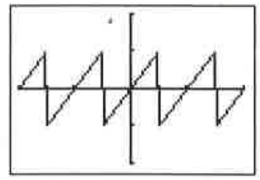
$[-2\pi, 2\pi]$ by $[-0.5\pi, 0.5\pi]$

(b) Domain all reals, range $[0, \pi]$, period 2π



$[-2\pi, 2\pi]$ by $[-0, \pi]$

(c) Domain all reals except $\pi/2 + n\pi$, n an integer, range $(-\pi/2, \pi/2)$, period π. Discontinuity is not removable.



$[-2\pi, 2\pi]$ by $[-\pi, \pi]$

67. $y = \dfrac{\pi}{2} - \tan^{-1} x$ **69.** $\dfrac{18}{\pi}\tan^{-1} x + 33$

SECTION 4.8

Exploration 1

1. Unit circle **3.** Since the grapher is plotting points along the unit circle, it covers the circle at a constant speed. Toward the extremes its motion is mostly vertical, so not much horizontal progress (which is all that we see) occurs. Toward the middle, the motion is mostly horizontal, so it moves faster.

Quick Review 4.8

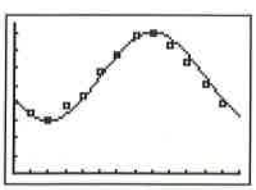
1. $b = 15 \cot 31° \approx 24.964$, $c = 15 \csc 31° \approx 29.124$ **3.** $b = 28 \cot 28° - 28 \cot 44° \approx 23.665$, $c = 28 \csc 28° \approx 59.642$, $a = 28 \csc 44° \approx 40.308$ **5.** Complement: 58°, supplement: 148° **7.** 45° **9.** Amplitude: 3; period: π

Exercises 4.8

All triangles in the supplied figures are right triangles.

1. $300\sqrt{3} \approx 519.62$ ft **3.** $120 \cot 10° \approx 680.55$ ft **5.** wire length $= 5 \sec 80° \approx 28.79$ ft; tower height $= 5 \tan 80° \approx 28.36$ ft **7.** $185 \tan 80°1'12'' \approx 1051$ ft **9.** $100 \tan 83°12' \approx 839$ ft **11.** $10 \tan 55° \approx 14.3$ ft **13.** $4.25 \tan 35° \approx 2.98$ mi **15.** $200(\tan 40° - \tan 30°) \approx 52.35$ ft **17.** distance: $60\sqrt{2} < 84.853$ naut mi; bearing is 140°. **19.** $1097 \cot 19° \approx 3186$ ft **21.** $325 \tan 63° \approx 638$ ft **23.** $36.5 \tan 15° \approx 9.8$ ft **25.** $\dfrac{550}{\cot 70° - \cot 80°} \approx 2931$ ft **27.** **(a)** 8 cycles/sec **(b)** $d = 6 \cos 16\pi t$ **(c)** about 4.1 in. left of the starting position **29.** $d = 3 \cos 4\pi t$ cm **31.** **(a)** 25 ft **(b)** 33 ft **(c)** $\pi/10$ radians/sec **33.** **(a)** $\pi/6$ **(b)** The amplitude is half this difference (the maximum minus the minimum): $a = 25°$. **(c)** 55° **(d)** $h = 5$; $y = 25 \sin\left[\dfrac{\pi}{6}(t - 5)\right] + 55$

(e)



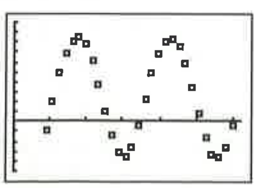
$[0, 13]$ by $[0, 85]$

(f) $y = 70$ when $t \approx 6.23$ (about July 8) or $t \approx 9.77$ (about October 24).

35. True. Since the frequency and the period are reciprocals, the higher the frequency the shorter the period.

37. (d) **39.** (d)

41. **(a)**



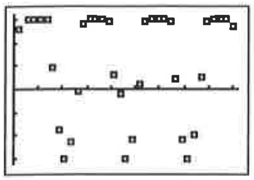
$[0, 0.0062]$ by $[-0.5, 1]$

(b) The first is the best. **(c)** About $\dfrac{2464}{2\pi} = \dfrac{1232}{\pi} \approx 392$ oscillations/sec

43. $2.5 \cot \dfrac{\pi}{7} \approx 5.2$ cm **45.** $AC \approx 33.6$ in.; $BD \approx 12.9$ in.

47. $\tan^{-1} 0.06 \approx 3.4°$

49. (a)



[0, 0.0092] by [−1.6, 1.6]

(b) One pretty good match is $y = 1.51971 \sin[2467(t - 0.0002)]$ (that is, $a = 1.51971$, $b = 2467$, $h = 0.0002$). Answers will vary but should be close to these values.

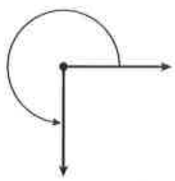
(c) Frequency: about $\frac{2467}{2\pi} \approx 393$ Hz. It appears to be a G.

(d) G

CHAPTER 4 REVIEW EXERCISES

1. positive y-axis; 450° **3.** III; $-\frac{3\pi}{4}$ **5.** I; $\frac{13\pi}{30}$ **7.** I; 15°

9. 270° or $\frac{3\pi}{2}$ radians



11. $30° = \frac{\pi}{6}$ radians **13.** $120° = \frac{2\pi}{3}$ radians

15. $360° + \tan^{-1}(-2) \approx 296.565° \approx 5.176$ radians

17. $\frac{1}{2}$ **19.** 1 **21.** $\frac{1}{2}$ **23.** 2 **25.** −1 **27.** 0

29. $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$, $\csc\left(-\frac{\pi}{6}\right) = -2$, $\sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$, $\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

31. $\sin(-135°) = -\frac{1}{\sqrt{2}}$, $\cos(-135°) = -\frac{1}{\sqrt{2}}$, $\tan(-135°) = 1$, $\csc(-135°) = -\sqrt{2}$, $\sec(-135°) = -\sqrt{2}$, $\cot(-135°) = 1$

33. $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\tan\alpha = \frac{5}{12}$, $\csc\alpha = \frac{13}{5}$, $\sec\alpha = \frac{13}{12}$, $\cot\alpha = \frac{12}{5}$

35. $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$, $\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$ **37.** ≈ 4.075 radians

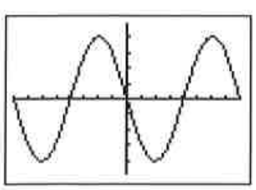
39. $a = 15 \sin 35° \approx 8.604$, $b = 15 \cos 35° \approx 12.287$, $\beta = 55°$ **41.** $b = 7 \tan 48° \approx 7.774$, $c = \frac{7}{\cos 48°} \approx 10.461$, $\alpha = 42°$

43. $a = 2\sqrt{6} \approx 4.90$, $\alpha \approx 44.42°$, $\beta \approx 45.58°$ **45.** III **47.** II

49. $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = -2$, $\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = -\frac{1}{2}$

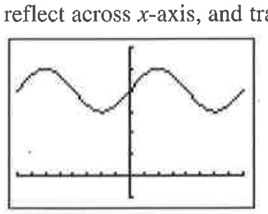
51. $\sin\theta = -\frac{3}{\sqrt{34}}$, $\cos\theta = -\frac{5}{\sqrt{34}}$, $\tan\theta = \frac{3}{5}$, $\csc\theta = -\frac{\sqrt{34}}{3}$, $\sec\theta = -\frac{\sqrt{34}}{5}$, $\cot\theta = \frac{5}{3}$

53. Starting from $y = \sin x$, translate left π units.



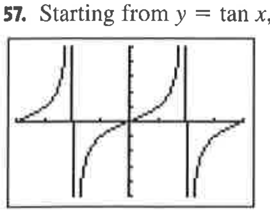
[−2π, 2π] by [−1.2, 1.2]

55. Starting from $y = \cos x$, translate left $\frac{\pi}{2}$ units, reflect across x-axis, and translate up 4 units.



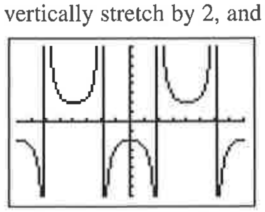
[−2π, 2π] by [−1, 6]

57. Starting from $y = \tan x$, horizontally shrink by $\frac{1}{2}$.



[−0.5π, 0.5π] by [−5, 5]

59. Starting from $y = \sec x$, horizontally stretch by 2, vertically stretch by 2, and reflect across x-axis (in any order).



[−4π, 4π] by [−8, 8]

61. Amplitude: 2; period: $\frac{2\pi}{3}$; phase shift: 0; domain: $(-\infty, \infty)$; range: $[-2, 2]$

63. Amplitude: 1.5; period: π; phase shift: $\frac{\pi}{8}$; domain: $(-\infty, \infty)$; range: $[-1.5, 1.5]$

65. Amplitude: 4; period: π; phase shift: $\frac{1}{2}$; domain: $(-\infty, \infty)$; range: $[-4, 4]$

67. $a \approx 4.47$, $b = 1$, and $h \approx 1.11$ **69.** $\approx 49.996° \approx 0.873$ radians **71.** $45° = \pi/4$ rad

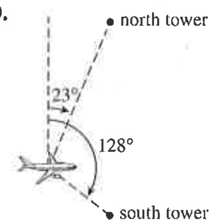
73. Starting from $y = \sin^{-1} x$, horizontally shrink by $\frac{1}{3}$. Domain: $\left[-\frac{1}{3}, \frac{1}{3}\right]$; range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **75.** Starting from $y = \sin^{-1} x$, translate right 1 unit, horizontally shrink by $\frac{1}{3}$, translate up 2 units. Domain: $\left[0, \frac{2}{3}\right]$; range: $\left[2 - \frac{\pi}{2}, 2 + \frac{\pi}{2}\right]$

77. $5\pi/6$ **79.** $3\pi/4$ **81.** $3\pi/2$ **85.** 1 **87.** 3/4 **83.** As $|x| \to \infty$, $\frac{\sin x}{x^2} \to 0$. **85.** 1 **87.** 3/4

89. Periodic; period: π; domain: $x \neq \frac{\pi}{2} + n\pi$, n an integer; range: $[1, \infty)$

91. Not periodic; domain: $x \neq \frac{\pi}{2} + n\pi$, n an integer; range: $(-\infty, \infty)$ **93.** $4\pi/3$ **95.** $100 \tan 78° \approx 470$ m

97. $150(\cot 18° - \cot 42°) \approx 295$ ft **99.**



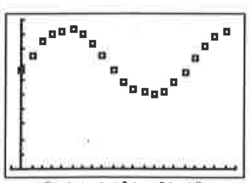
101. $62 \tan 72°24' \approx 195.4$ ft **103.** $22\pi/15 \approx 4.6$ in.

105. $x = 112$ and 272, which correspond to April 22 and Sept 29.

Chapter 4 Project

Answers are based on the sample data shown in the table.

1.



$[-0.1, 2.1]$ by $[0, 1]$

3. The constant a represents half the distance the pendulum bob swings as it moves from its highest point to its lowest point; k represents the distance from the detector to the pendulum bob when it is in mid-swing.

5. $y \approx 0.22 \sin(3.87x - 0.16) + 0.71$; Most calculator/computer regression models are expressed in the form $y = a \sin(bx + p) + k$, where $-p/b = h$ in the equation $y = a \sin(b(x - h)) + k$. The latter equation form differs from $y = a \cos(b(x - h)) + k$ only in h.

SECTION 5.1

Exploration 1

1. $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **3.** $\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Quick Review 5.1

1. 1.1760 rad = 67.380° **3.** 2.4981 rad = 143.130° **5.** $(a - b)^2$ **7.** $(2x + y)(x - 2y)$ **9.** $\frac{y - 2x}{xy}$ **11.** xy

Exercises 5.1

1. $\sin\theta = 3/5$ and $\cos\theta = 4/5$ **3.** $\tan\theta = -\sqrt{15}$ and $\cot = -1/\sqrt{15} = -\sqrt{15}/15$ **5.** 0.45 **7.** −0.73 **9.** $\sin x$ **11.** 1 **13.** $\tan^2 x$ **15.** $\cos x \sin^2 x$ **17.** −1 **19.** −1 **21.** 1 **23.** $\cos x$ **25.** 2 **27.** $\sec x$ **29.** $\tan x$ **31.** $\tan x$ **33.** $2\csc^2 x$ **35.** $-\sin x$ **37.** $\cot x$ **39.** $(\cos x + 1)^2$ **41.** $(1 - \sin x)^2$ **43.** $(\cos x + 1)(2\cos x - 1)$ **45.** $(2\tan x - 1)^2$ **47.** $1 - \sin x$ **49.** $1 - \cos x$ **51.** $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ **53.** $\{0, \pi\}$ **55.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$