61. Amplitude: 2; period: $\frac{2\pi}{3}$; phase shift: 0; domain: $(-\infty, \infty)$; range: $[-2, 2]$

63. Amplitude: 1.5; period: π; phase shift: $\frac{\pi}{8}$; domain: $(-\infty, \infty)$; range: $[-1.5, 1.5]$

65. Amplitude: 4; period: π; phase shift: $\frac{1}{2}$; domain: $(-\infty, \infty)$; range: $[-4, 4]$

67. $a \approx 4.47$, $b = 1$, and $h \approx 1.11$ **69.** $\approx 49.996° \approx 0.873$ radians **71.** $45° = \pi/4$ rad

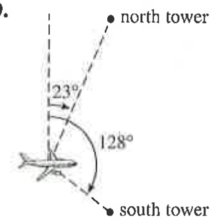
73. Starting from $y = \sin^{-1} x$, horizontally shrink by $\frac{1}{3}$. Domain: $\left[-\frac{1}{3}, \frac{1}{3}\right]$; range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **75.** Starting from $y = \sin^{-1} x$, translate right 1 unit, horizontally shrink by $\frac{1}{3}$, translate up 2 units. Domain: $\left[0, \frac{2}{3}\right]$; range: $\left[2 - \frac{\pi}{2}, 2 + \frac{\pi}{2}\right]$

77. $5\pi/6$ **79.** $3\pi/4$ **81.** $3\pi/2$ **85.** 1 **87.** 3/4 **83.** As $|x| \to \infty$, $\frac{\sin x}{x^2} \to 0$. **85.** 1 **87.** 3/4

89. Periodic; period: π; domain: $x \neq \frac{\pi}{2} + n\pi$, n an integer; range: $[1, \infty)$

91. Not periodic; domain: $x \neq \frac{\pi}{2} + n\pi$, n an integer; range: $(-\infty, \infty)$ **93.** $4\pi/3$ **95.** $100 \tan 78° \approx 470$ m

97. $150(\cot 18° - \cot 42°) \approx 295$ ft **99.**



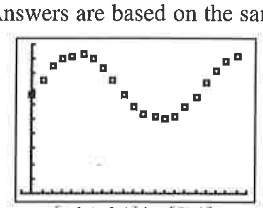
101. $62 \tan 72°24' \approx 195.4$ ft **103.** $22\pi/15 \approx 4.6$ in.

105. $x = 112$ and 272, which correspond to April 22 and Sept 29.

Chapter 4 Project

Answers are based on the sample data shown in the table.

1.



$[-0.1, 2.1]$ by $[0, 1]$

3. The constant a represents half the distance the pendulum bob swings as it moves from its highest point to its lowest point; k represents the distance from the detector to the pendulum bob when it is in mid-swing.

5. $y \approx 0.22 \sin(3.87x - 0.16) + 0.71$; Most calculator/computer regression models are expressed in the form $y = a \sin(bx + p) + k$, where $-p/b = h$ in the equation $y = a \sin(b(x - h)) + k$. The latter equation form differs from $y = a \cos(b(x - h)) + k$ only in h.

SECTION 5.1

Exploration 1

1. $\cos \theta = \frac{1}{\sec \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, and $\tan \theta = \frac{\sin \theta}{\cos \theta}$ **3.** $\csc \theta = \frac{1}{\sin \theta}$, $\cot \theta = \frac{1}{\tan \theta}$, and $\cot \theta = \frac{\cos \theta}{\sin \theta}$

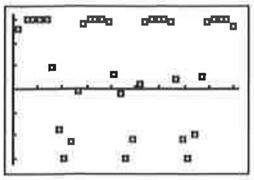
Quick Review 5.1

1. 1.1760 rad = 67.380° **3.** 2.4981 rad = 143.130° **5.** $(a - b)^2$ **7.** $(2x + y)(x - 2y)$ **9.** $\frac{y - 2x}{xy}$ **11.** xy

Exercises 5.1

1. $\sin \theta = 3/5$ and $\cos \theta = 4/5$ **3.** $\tan \theta = -\sqrt{15}$ and $\cot = -1/\sqrt{15} = -\sqrt{15}/15$ **5.** 0.45 **7.** -0.73 **9.** $\sin x$ **11.** 1 **13.** $\tan^2 x$ **15.** $\cos x \sin^2 x$ **17.** -1 **19.** -1 **21.** 1 **23.** $\cos x$ **25.** 2 **27.** $\sec x$ **29.** $\tan x$ **31.** $\tan x$ **33.** $2\csc^2 x$ **35.** $-\sin x$ **37.** $\cot x$ **39.** $(\cos x + 1)^2$ **41.** $(1 - \sin x)^2$ **43.** $(\cos x + 1)(2\cos x - 1)$ **45.** $(2\tan x - 1)^2$ **47.** $1 - \sin x$ **49.** $1 - \cos x$ **51.** $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ **53.** $\{0, \pi\}$ **55.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

49. (a)



[0, 0.0092] by [−1.6, 1.6]

(b) One pretty good match is $y = 1.51971\sin[2467(t - 0.0002)]$ (that is, $a = 1.51971$, $b = 2467$, $h = 0.0002$). Answers will vary but should be close to these values.

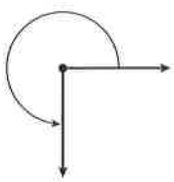
(c) Frequency: about $\frac{2467}{2\pi} \approx 393$ Hz. It appears to be a G.

(d) G

CHAPTER 4 REVIEW EXERCISES

1. positive y-axis; 450° **3.** III; $-\frac{3\pi}{4}$ **5.** I; $\frac{13\pi}{30}$ **7.** I; 15°

9. 270° or $\frac{3\pi}{2}$ radians



11. $30° = \frac{\pi}{6}$ radians **13.** $120° = \frac{2\pi}{3}$ radians

15. $360° + \tan^{-1}(-2) \approx 296.565° \approx 5.176$ radians

17. $\frac{1}{2}$ **19.** 1 **21.** $\frac{1}{2}$ **23.** 2 **25.** −1 **27.** 0

29. $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$, $\csc\left(-\frac{\pi}{6}\right) = -2$, $\sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$, $\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

31. $\sin(-135°) = -\frac{1}{\sqrt{2}}$, $\cos(-135°) = -\frac{1}{\sqrt{2}}$, $\tan(-135°) = 1$, $\csc(-135°) = -\sqrt{2}$, $\sec(-135°) = -\sqrt{2}$, $\cot(-135°) = 1$

33. $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\tan\alpha = \frac{5}{12}$, $\csc\alpha = \frac{13}{5}$, $\sec\alpha = \frac{13}{12}$, $\cot\alpha = \frac{12}{5}$

35. $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$, $\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$ **37.** ≈ 4.075 radians

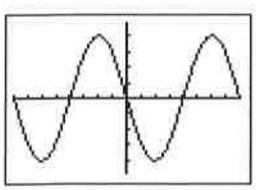
39. $a = 15\sin 35° \approx 8.604$, $b = 15\cos 35° \approx 12.287$, $\beta = 55°$ **41.** $b = 7\tan 48° \approx 7.774$, $c = \frac{7}{\cos 48°} \approx 10.461$, $\alpha = 42°$

43. $a = 2\sqrt{6} \approx 4.90$, $\alpha \approx 44.42°$, $\beta \approx 45.58°$ **45.** III **47.** II

49. $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = -2$, $\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = -\frac{1}{2}$

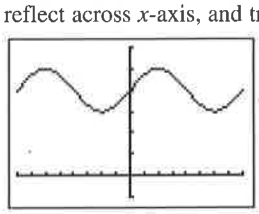
51. $\sin\theta = -\frac{3}{\sqrt{34}}$, $\cos\theta = -\frac{5}{\sqrt{34}}$, $\tan\theta = \frac{3}{5}$, $\csc\theta = -\frac{\sqrt{34}}{3}$, $\sec\theta = -\frac{\sqrt{34}}{5}$, $\cot\theta = \frac{5}{3}$

53. Starting from $y = \sin x$, translate left π units.



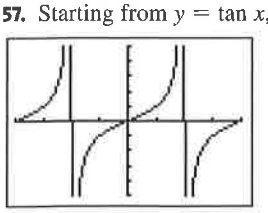
[-2π, 2π] by [−1.2, 1.2]

55. Starting from $y = \cos x$, translate left $\frac{\pi}{2}$ units, reflect across x-axis, and translate up 4 units.



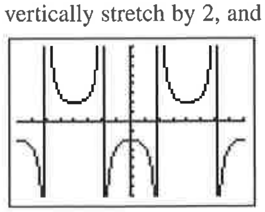
[-2π, 2π] by [−1, 6]

57. Starting from $y = \tan x$, horizontally shrink by $\frac{1}{2}$.



[-0.5π, 0.5π] by [−5, 5]

59. Starting from $y = \sec x$, horizontally stretch by 2, vertically stretch by 2, and reflect across x-axis (in any order).



[-4π, 4π] by [−8, 8]

57. $\pm\frac{\pi}{3} + 2n\pi, n = 0, \pm 1, \pm 2, \ldots$ **59.** $n\pi, n = 0, \pm 1, \pm 2, \ldots$ **61.** $n\pi, n = 0, \pm 1, \pm 2, \ldots$

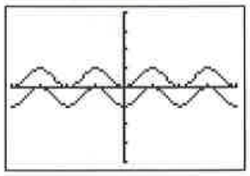
63. $\{\pm 1.1918 + 2n\pi \mid n = 0, \pm 1, \pm 2, \ldots\}$ **65.** $\{0.3047 + 2n\pi \text{ or } 2.8369 + 2n\pi \mid n = 0, \pm 1, \pm 2, \ldots\}$
67. $\{\pm 0.8861 + n\pi \mid n = 0, \pm 1, \pm 2, \ldots\}$ **69.** $|\sin\theta|$ **71.** $3|\tan\theta|$ **73.** $9|\sec\theta|$

75. True. Since secant is an even function, $\sec\left(x - \frac{\pi}{2}\right) = \sec\left(\frac{\pi}{2} - x\right)$, which equals $\csc x$ by one of the cofunction identities.

77. (d) **79.** (c)

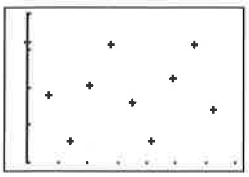
81. $\sin x$, $\cos x = \pm\sqrt{1 - \sin^2 x}$, $\tan x = \pm\frac{\sin x}{\sqrt{1 - \sin^2 x}}$, $\csc x = \frac{1}{\sin x}$, $\sec x = \pm\frac{1}{\sqrt{1 - \sin^2 x}}$, $\cot x = \pm\frac{\sqrt{1 - \sin^2 x}}{\sin x}$

83. The two functions are parallel to each other, separated by 1 unit for every x. At any x, the distance between the two graphs is $\sin^2 x - (-\cos^2 x) = \sin^2 x + \cos^2 x = 1$.



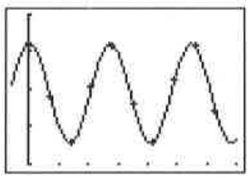
$[-2\pi, 2\pi]$ by $[-4, 4]$

85. **(a)**



$[-6, 70]$ by $[220{,}000, 260{,}000]$

(b) The equation is $y = 13111 \sin(0.22998\,x + 1.571) + 238856$.



$[-6, 70]$ by $[220{,}000, 260{,}000]$

(c) $(2\pi)/0.22998 \approx 27.32$ days. This is the number of days that it takes the moon to make one complete orbit of the Earth (known as the moon's sidereal period).
(d) 225,745 miles
(e) $y = 13{,}111\cos(-0.22998\,x) + 238{,}856$, $y = 13{,}111\cos(0.22998\,x) + 238{,}856$

87. Factor the left-hand side: $\sin^4\theta - \cos^4\theta = (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) = (\sin^2\theta - \cos^2\theta)\cdot 1 = \sin^2\theta - \cos^2\theta$

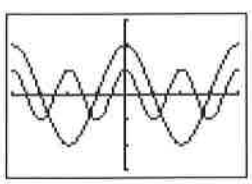
89. Use the hint:
$\sin(\pi - x) = \sin(\pi/2 - (x - \pi/2))$
$= \cos(x - \pi/2)$ Cofunction identity
$= \cos(\pi/2 - x)$ Since cos is even
$= \sin x$ Cofunction identity

91. Since A, B, and C are angles of a triangle, $A + B = \pi - C$. So: $\sin(A + B) = \sin(\pi - C) = \sin C$

SECTION 5.2

Exploration 1

1. The graphs lead us to conclude that this is not an identity.



$[-2\pi, 2\pi]$ by $[-4, 4]$

3. Yes.
5. No. The graph window cannot show the full graphs, so they could differ outside the viewing window. Also, the function values could be so close that the graphs *appear* to coincide.

Quick Review 5.2

1. $\frac{\sin x + \cos x}{\sin x \cos x}$ **3.** $\frac{1}{\sin x \cos x}$ **5.** 1 **7.** No. Any negative x. **9.** No. Any x for which $\sin x < 0$, e.g., $x = -\pi/2$.
11. Yes

Exercises 5.2

1. One possible proof: $\frac{x^3 - x^2}{x} - (x - 1)(x + 1) = \frac{x(x^2 - x)}{x} - (x^2 - 1) = x^2 - x - (x^2 - 1) = -x + 1 = 1 - x$

3. One possible proof: $\frac{x^2 - 4}{x - 2} - \frac{x^2 - 9}{x + 3} = \frac{(x + 2)(x - 2)}{x - 2} - \frac{(x + 3)(x - 3)}{x + 3} = x + 2 - (x - 3) = 5$

5. Yes. **7.** No. **9.** Yes. **11.** $(\cos x)(\tan x + \sin x \cot x) = \cos x \cdot \dfrac{\sin x}{\cos x} + \cos x \sin x \cdot \dfrac{\cos x}{\sin x} = \sin x + \cos^2 x$

13. $(1 - \tan x)^2 = 1 - 2\tan x + \tan^2 x = (1 + \tan^2 x) - 2\tan x = \sec^2 x - 2\tan x$

15. $\dfrac{(1-\cos u)(1+\cos u)}{\cos^2 u} = \dfrac{1-\cos^2 u}{\cos^2 u} = \dfrac{\sin^2 u}{\cos^2 u} = \tan^2 u$ **17.** $\dfrac{\cos^2 x - 1}{\cos x} = \dfrac{-\sin^2 x}{\cos x} = -\dfrac{\sin x}{\cos x} \cdot \sin x = -\tan x \sin x$

19. Multiply out the expression on the left side.

21. $(\cos t - \sin t)^2 + (\cos t + \sin t)^2 = \cos^2 t - 2\cos t \sin t + \sin^2 t + \cos^2 t + 2\cos t \sin t + \sin^2 t = 2\cos^2 t + 2\sin^2 t = 2$

23. $\dfrac{1+\tan^2 x}{\sin^2 x + \cos^2 x} = \dfrac{\sec^2 x}{1} = \sec^2 x$ **25.** $\dfrac{\cos\beta}{1+\sin\beta} = \dfrac{\cos^2\beta}{\cos\beta(1+\sin\beta)} = \dfrac{1-\sin^2\beta}{\cos\beta(1+\sin\beta)} = \dfrac{(1-\sin\beta)(1+\sin\beta)}{\cos\beta(1+\sin\beta)} = \dfrac{1-\sin\beta}{\cos\beta}$

27. $\dfrac{\tan^2 x}{\sec x + 1} = \dfrac{\sec^2 x - 1}{\sec x + 1} = \sec x - 1 = \dfrac{1}{\cos x} - 1 = \dfrac{1-\cos x}{\cos x}$

29. $\cot^2 x - \cos^2 x = \left(\dfrac{\cos x}{\sin x}\right)^2 - \cos^2 x = \dfrac{\cos^2 x(1-\sin^2 x)}{\sin^2 x} = \cos^2 x \cdot \dfrac{\cos^2 x}{\sin^2 x} = \cos^2 x \cot^2 x$

31. $\cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = 1(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

33. $(x\sin\alpha + y\cos\alpha)^2 + (x\cos\alpha - y\sin\alpha)^2 = (x^2\sin^2\alpha + 2xy\sin\alpha\cos\alpha + y^2\cos^2\alpha) + (x^2\cos^2\alpha - 2xy\cos\alpha\sin\alpha + y^2\sin^2\alpha)$
$= x^2\sin^2\alpha + y^2\cos^2\alpha + x^2\cos^2\alpha + y^2\sin^2\alpha = (x^2+y^2)(\sin^2\alpha + \cos^2\alpha) = x^2 + y^2$

35. $\dfrac{\tan x}{\sec x - 1} = \dfrac{\tan x(\sec x + 1)}{\sec^2 x - 1} = \dfrac{\tan x(\sec x + 1)}{\tan^2 x} = \dfrac{\sec x + 1}{\tan x}$

37. $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{(\sin x + \cos x)^2} = \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + 2\sin x\cos x + \cos^2 x} = \dfrac{\sin^2 x - (1-\sin^2 x)}{1 + 2\sin x\cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x\cos x}$

39. $\dfrac{\sin t}{1-\cos t} + \dfrac{1+\cos t}{\sin t} = \dfrac{\sin^2 t + (1+\cos t)(1-\cos t)}{(\sin t)(1-\cos t)} = \dfrac{1-\cos^2 t + 1 - \cos^2 t}{(\sin t)(1-\cos t)} = \dfrac{2(1-\cos^2 t)}{(\sin t)(1-\cos t)} = \dfrac{2(1+\cos t)}{\sin t}$

41. $\sin^2 x\cos^3 x = \sin^2 x\cos^2 x\cos x = \sin^2 x(1-\sin^2 x)(\cos x) = (\sin^2 x - \sin^4 x)(\cos x)$

43. $\cos^5 x = \cos^4 x\cos x = (\cos^2 x)^2\cos x = (1-\sin^2 x)^2(\cos x) = (1 - 2\sin^2 x + \sin^4 x)(\cos x)$

45. $\dfrac{\tan x}{1-\cot x} + \dfrac{\cot x}{1-\tan x} = \dfrac{\tan x}{1-\cot x}\cdot\dfrac{\sin x}{\sin x} + \dfrac{\cot x}{1-\tan x}\cdot\dfrac{\cos x}{\cos x} = \left(\dfrac{\sin^2 x/\cos x}{\sin x - \cos x} + \dfrac{\cos^2 x/\sin x}{\cos x - \sin x}\right)\dfrac{\sin x\cos x}{\sin x\cos x}$
$= \dfrac{\sin^3 x - \cos^3 x}{\sin x\cos x(\sin x - \cos x)} = \dfrac{\sin^2 x + \sin x\cos x + \cos^2 x}{\sin x\cos x} = \dfrac{1 + \sin x\cos x}{\sin x\cos x} = \dfrac{1}{\sin x\cos x} + 1 = \csc x\sec x + 1$

47. $\dfrac{2\tan x}{1-\tan^2 x} + \dfrac{1}{2\cos^2 x - 1} = \dfrac{2\tan x}{1-\tan^2 x}\cdot\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{1}{\cos^2 x - \sin^2 x} = \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x} + \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} = \dfrac{2\sin x\cos x + \cos^2 x + \sin^2 x}{(\cos x - \sin x)(\cos x + \sin x)}$
$= \dfrac{(\cos x + \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$ **49.** $\cos^3 x = (\cos^2 x)(\cos x) = (1-\sin^2 x)(\cos x)$

51. $\sin^5 x = (\sin^4 x)(\sin x) = (\sin^2 x)^2(\sin x) = (1-\cos^2 x)^2(\sin x) = (1 - 2\cos^2 x + \cos^4 x)(\sin x)$

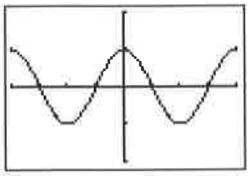
53. (d) **55.** (c) **57.** (b) **59.** True. If x is in the domain of both sides of the equation, then $x \geq 0$. The equation holds for all $x \geq 0$, so it is an identity. **61.** (e) **63.** (b) **65.** $\sin x$ **67.** 1 **69.** 1

71. If A and B are complementary angles, then $\sin^2 A + \sin^2 B = \sin^2 A + \sin^2(\pi/2 - A) = \sin^2 A + \cos^2 A = 1$

73. Multiply and divide by $1 - \sin t$ under the radical: $\sqrt{\dfrac{1-\sin t}{1+\sin t}\cdot\dfrac{1-\sin t}{1-\sin t}} = \sqrt{\dfrac{(1-\sin t)^2}{1-\sin^2 t}} = \sqrt{\dfrac{(1-\sin t)^2}{\cos^2 t}} = \dfrac{|1 - \sin t|}{|\cos t|}$
since $\sqrt{a^2} = |a|$. Now, since $1 - \sin t \geq 0$, we can dispense with the absolute value in the numerator, but it must stay in the denominator.

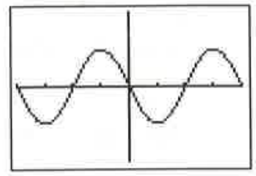
75. $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + \cos^6 x = (1-\cos^2 x)^3 + \cos^6 x = (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) + \cos^6 x$
$= 1 - 3\cos^2 x(1-\cos^2 x) = 1 - 3\cos^2 x\sin^2 x$ **77.** $\ln|\tan x| = \ln\dfrac{|\sin x|}{|\cos x|} = \ln|\sin x| - \ln|\cos x|$

79. (a) They are not equal. Shown is the window $[-2\pi, 2\pi]$ by $[-2, 2]$; graphing on nearly any viewing window does not show any apparent difference—but using TRACE, one finds that the y coordinates are not identical. Likewise, a table of values will show slight differences; for example, when $x = 1$, $y_1 = 0.53988$ while $y_2 = 0.54030$.



$[-2\pi, 2\pi]$ by $[-2, 2]$

(b) One choice for h is 0.001 (shown). The function y_3 is a combination of three sinusoidal functions ($1000\sin(x + 0.001)$, $1000 \sin x$, and $\cos x$), all with period 2π.



$[-2\pi, 2\pi]$ by $[-0.001, 0.001]$

81. In the decimal window, the x coordinates used to plot the graph on the calculator are (e.g.) 0, 0.1, 0.2, 0.3, etc. — that is, $x = n/10$, where n is an integer. Then $10\pi x = \pi n$, and the sine of integer multiples of π is 0; therefore, $\cos x + \sin 10\pi x = \cos x + \sin \pi n = \cos x + 0 = \cos x$. However, for other choices of x, such as $x = \frac{1}{\pi}$, we have $\cos x + \sin 10\pi x = \cos x + \sin 10 \neq \cos x$.

SECTION 5.3

Exploration 1

1. No **3.** $\tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right) = -\sqrt{3}$, $\tan\frac{\pi}{3} + \tan\frac{\pi}{3} = 2\sqrt{3}$. (Many other answers are possible.)

Quick Review 5.3

1. $45° - 30°$ **3.** $210° - 45°$ **5.** $\frac{2\pi}{3} - \frac{\pi}{4}$ **7.** No **9.** Yes

Exercises 5.3

1. $\frac{\sqrt{6} - \sqrt{2}}{4}$ **3.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **5.** $\frac{\sqrt{2} + \sqrt{6}}{4}$ **7.** $2 + \sqrt{3}$ **9.** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **11.** $\sin 25°$ **13.** $\sin\frac{7\pi}{10}$ **15.** $\tan 66°$

17. $\cos\left(x - \frac{\pi}{7}\right)$ **19.** $\sin 2x$ **21.** $\tan(2y + 3x)$ **23.** $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2} = \sin x \cdot 0 - \cos x \cdot 1 = -\cos x$

25. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} = \cos x \cdot 0 + \sin x \cdot 1 = \sin x$

27. $\sin\left(x + \frac{\pi}{6}\right) = \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} = \sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2}$

29. $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\tan\theta + \tan(\pi/4)}{1 - \tan\theta\tan(\pi/4)} = \frac{\tan\theta + 1}{1 - \tan\theta \cdot 1} = \frac{1 + \tan\theta}{1 - \tan\theta}$ **31.** Equations (b) and (f) **33.** Equations (d) and (h)

35. $x = n\pi, n = 0, \pm 1, \pm 2, \ldots$ **37.** $\sin\left(\frac{\pi}{2} - u\right) = \sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u = 1 \cdot \cos u - 0 \cdot \sin u = \cos u$

39. $\cot\left(\frac{\pi}{2} - u\right) = \frac{\cos(\pi/2 - u)}{\sin(\pi/2 - u)} = \frac{\sin u}{\cos u} = \tan u$, using the first two cofunction identities.

41. $\csc\left(\frac{\pi}{2} - u\right) = \frac{1}{\sin(\pi/2 - u)} = \frac{1}{\cos u} = \sec u$, using the second cofunction identity.

43. $y \approx 5\sin(x + 0.9273)$ **45.** $y \approx 2.236\sin(3x + 0.4636)$

47. $\sin(x - y) + \sin(x + y) = (\sin x \cos y - \cos x \sin y) + (\sin x \cos y + \cos x \sin y) = 2 \sin x \cos y$

49. $\cos 3x = \cos[(x + x) + x] = \cos(x + x)\cos x - \sin(x + x)\sin x = (\cos x \cos x - \sin x \sin x)\cos x - (\sin x \cos x + \cos x \sin x)\sin x = \cos^3 x - \sin^2 x \cos x - 2\cos x \sin^2 x = \cos^3 x - 3\sin^2 x \cos x$

51. $\cos 3x + \cos x = \cos(2x + x) + \cos(2x - x)$; use Exercise 48 with x replaced with $2x$ and y replaced with x.

53. $\tan(x + y)\tan(x - y) = \left(\frac{\tan x + \tan y}{1 - \tan x \tan y}\right)\left(\frac{\tan x - \tan y}{1 + \tan x \tan y}\right) = \frac{\tan^2 x - \tan^2 y}{1 - \tan^2 x \tan^2 y}$ since both the numerator and denominator are factored forms for differences of squares.

55. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \dfrac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} \cdot \dfrac{1/(\cos x \cos y)}{1/(\cos x \cos y)}$

$= \dfrac{(\sin x \cos y)/(\cos x \cos y) + (\cos x \sin y)/(\cos x \cos y)}{(\sin x \cos y)/(\cos x \cos y) - (\cos x \sin y)/(\cos x \cos y)} = \dfrac{(\sin x/\cos x) + (\sin y/\cos y)}{(\sin x/\cos x) - (\sin y/\cos y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$

57. False. For example, $\cos 3\pi + \cos 4\pi = 0$, but 3π and 4π are not supplementary. **59.** (a) **61.** (b)

63. $\tan(u-v) = \dfrac{\sin(u-v)}{\cos(u-v)} = \dfrac{\sin u \cos v - \cos u \sin v}{\cos u \cos v + \sin u \sin v} = \dfrac{\dfrac{\sin u \cos v}{\cos u \cos v} - \dfrac{\cos u \sin v}{\cos u \cos v}}{\dfrac{\cos u \cos v}{\cos u \cos v} + \dfrac{\sin u \sin v}{\cos u \cos v}} = \dfrac{\dfrac{\sin u}{\cos u} - \dfrac{\sin v}{\cos v}}{1 + \dfrac{\sin u \sin v}{\cos u \cos v}} = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

65. The identity would involve $\tan\left(\dfrac{3\pi}{2}\right)$, which does not exist. $\tan\left(x - \dfrac{3\pi}{2}\right) = \dfrac{\sin\left(x - \dfrac{3\pi}{2}\right)}{\cos\left(x - \dfrac{3\pi}{2}\right)} = \dfrac{\sin x \cos \dfrac{3\pi}{2} - \cos x \sin \dfrac{3\pi}{2}}{\cos x \cos \dfrac{3\pi}{2} - \sin x \sin \dfrac{3\pi}{2}}$

$= \dfrac{\sin x \cdot 0 - \cos x \cdot (-1)}{\cos x \cdot 0 + \sin x \cdot (-1)} = -\cot x$

67. $\dfrac{\cos(x+h) - \cos x}{h} = \dfrac{\cos x \cos h - \sin x \sin h - \cos x}{h} = \dfrac{\cos x(\cos h - 1) - \sin x \sin h}{h} = \cos x\left(\dfrac{\cos h - 1}{h}\right) - \sin x \dfrac{\sin h}{h}$

69. $\sin(A+B) = \sin(\pi - C) = \sin \pi \cos C - \cos \pi \sin C = 0 \cdot \cos C - (-1) \sin C = \sin C$

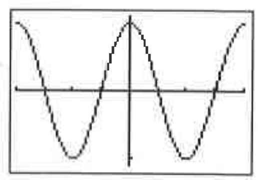
71. $\tan A + \tan B + \tan C = \dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B} + \dfrac{\sin C}{\cos C} = \dfrac{\sin A(\cos B \cos C) + \sin B(\cos A \cos C)}{\cos A \cos B \cos C} + \dfrac{\sin C(\cos A \cos B)}{\cos A \cos B \cos C}$

$= \dfrac{\cos C(\sin A \cos B + \cos A \sin B) + \sin C(\cos A \cos B)}{\cos A \cos B \cos C} = \dfrac{\cos C \sin(A+B) + \sin C(\cos(A+B) + \sin A \sin B)}{\cos A \cos B \cos C}$

$= \dfrac{\cos C \sin(\pi - C) + \sin C(\cos(\pi - C) + \sin A \sin B)}{\cos A \cos B \cos C} = \dfrac{\cos C \sin C + \sin C(-\cos C) + \sin C \sin A \sin B}{\cos A \cos B \cos C}$

$= \dfrac{\sin A \sin B \sin C}{\cos A \cos B \cos C} = \tan A \tan B \tan C$

73. This equation is easier to deal with after rewriting it as $\cos 5x \cos 4x + \sin 5x \sin 4x = 0$. The left side of this equation is the expanded form of $\cos(5x - 4x)$, which of course equals $\cos x$; the graph shown is simply $y = \cos x$. The equation $\cos x = 0$ is easily solved on the interval $[-2\pi, 2\pi]$: $x = \pm\dfrac{\pi}{2}$ or $x = \pm\dfrac{3\pi}{2}$. The original graph is so crowded that one cannot see where crossings occur. The window shown is $[-2\pi, 2\pi]$ by $[-1.1, 1.1]$.



$[-2\pi, 2\pi]$ by $[-1.1, 1.1]$

75. $B = B_{in} + B_{ref} = \dfrac{E_0}{c}\cos\left(\omega t - \dfrac{\omega x}{c}\right) + \dfrac{E_0}{c}\cos\left(\omega t + \dfrac{\omega x}{c}\right) = \dfrac{E_0}{c}\left[\cos\left(\omega t - \dfrac{\omega x}{c}\right) + \cos\left(\omega t + \dfrac{\omega x}{c}\right)\right]$

$= \dfrac{E_0}{c}\left(2 \cos \omega t \cos \dfrac{\omega x}{c}\right) = 2\dfrac{E_0}{c} \cos \omega t \cos \dfrac{\omega x}{c}$

The next-to-last step follows by the identity in Exercise 48.

SECTION 5.4

Exploration 1

1. $\sin^2 \dfrac{\pi}{8} = \dfrac{1 - \cos(\pi/4)}{2} = \dfrac{1 - (\sqrt{2}/2)}{2} \cdot \dfrac{2}{2} = \dfrac{2 - \sqrt{2}}{4}$ **3.** $\sin^2 \dfrac{9\pi}{8} = \dfrac{1 - \cos(9\pi/4)}{2} = \dfrac{1 - (\sqrt{2}/2)}{2} \cdot \dfrac{2}{2} = \dfrac{2 - \sqrt{2}}{4}$

Quick Review 5.4

1. $x = \frac{\pi}{4} + n\pi$, n an integer **3.** $x = \frac{\pi}{2} + n\pi$, n an integer. **5.** $x = -\frac{\pi}{4} + n\pi$, n an integer

7. $x = \frac{\pi}{6} + 2n\pi$ or $x = \frac{5\pi}{6} + 2n\pi$ or $x = \pm\frac{2\pi}{3} + 2n\pi$, n an integer. **9.** $A = (2)(3) + \frac{1}{2}(1)(3) + \frac{1}{2}(2)(3) = 10.5$ square units

Exercises 5.4

1. $\cos 2u = \cos(u + u) = \cos u \cos u - \sin u \sin u = \cos^2 u - \sin^2 u$

3. Starting with the result of Exercise 1: $\cos 2u = \cos^2 u - \sin^2 u = (1 - \sin^2 u) - \sin^2 u = 1 - 2\sin^2 u$

5. $0, \pi$ **7.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **9.** $0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$. **11.** $2\sin\theta\cos\theta + \cos\theta$ or $(\cos\theta)(2\sin\theta + 1)$

13. $2\sin\theta\cos\theta + 4\cos^3\theta - 3\cos\theta$ or $2\sin\theta\cos\theta + \cos^3\theta - 3\sin^2\theta\cos\theta$

15. $\sin 4x = \sin 2(2x) = 2\sin 2x \cos 2x$ **17.** $2\csc 2x = \frac{2}{\sin 2x} = \frac{2}{2\sin x\cos x} = \frac{1}{\sin^2 x}\cdot\frac{\sin x}{\cos x} = \csc^2 x \tan x$

19. $\sin 3x = \sin 2x \cos x + \cos 2x \sin x = 2\sin x\cos^2 x + (2\cos^2 x - 1)\sin x = (\sin x)(4\cos^2 x - 1)$

21. $\cos 4x = \cos 2(2x) = 1 - 2\sin^2 2x = 1 - 2(2\sin x\cos x)^2 = 1 - 8\sin^2 x\cos^2 x$ **23.** $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

25. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ **27.** $0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$ **29.** $\frac{\pi}{2}, \frac{3\pi}{2}, 0.1\pi, 0.9\pi, 1.3\pi, 1.7\pi$

31. $\frac{1}{2}\sqrt{2 - \sqrt{3}}$ **33.** $\frac{1}{2}\sqrt{2 - \sqrt{3}}$ **35.** $-2 - \sqrt{3}$

37. (a) Starting from the right side: $\frac{1}{2}(1 - \cos 2u) = \frac{1}{2}[1 - (1 - 2\sin^2 u)] = \frac{1}{2}(2\sin^2 u) = \sin^2 u$.

(b) Starting from the right side: $\frac{1}{2}(1 + \cos 2u) = \frac{1}{2}[1 + (2\cos^2 u - 1)] = \frac{1}{2}(2\cos^2 u) = \cos^2 u$.

39. $\sin^4 x = (\sin^2 x)^2 = \left[\frac{1}{2}(1 - \cos 2x)\right]^2 = \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x) = \frac{1}{4}\left[1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right]$
$= \frac{1}{8}(2 - 4\cos 2x + 1 + \cos 4x) = \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$

41. $\sin^3 2x = \sin 2x \sin^2 2x = \sin 2x \cdot \frac{1}{2}(1 - \cos 4x) = \frac{1}{2}(\sin 2x)(1 - \cos 4x)$

43. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$. General solution: $x = \pi + 2n\pi$, n an integer, or $x = \pm\frac{\pi}{3} + 2n\pi$, n an integer.

45. $0, \frac{\pi}{2}$. General solution: $x = \frac{\pi}{2} + 2n\pi$, n an integer, or $x = 2\pi n$, n an integer. **47.** False. For example, $f(x) = 2\sin x$ has period 2π and $g(x) = \cos x$ has period 2π, but the product $f(x)g(x) = 2\sin x\cos x = \sin 2x$ has period π. **49.** (d) **51.** (e)

53. (a) In the figure, the triangle with side lengths $x/2$ and R is a right triangle, since R is given as the perpendicular distance. Then the tangent of the angle $\theta/2$ is the ratio "opposite over adjacent": $\tan\frac{\theta}{2} = \frac{x/2}{R}$. Solving for x gives the desired equation. The central angle θ is $2\pi/n$ since one full revolution of 2π radians is divided evenly into n sections.

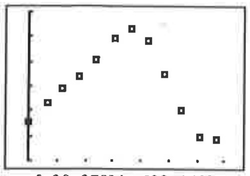
(b) $5.87 \approx 2R\tan\frac{\theta}{2}$, where $\theta = \frac{2\pi}{11}$, so $R \approx 5.87/\left(2\tan\frac{\pi}{11}\right) \approx 9.9957$, $R = 10$.

55. $\theta = \frac{\pi}{6}$; the maximum value is about 12.99 ft^3. **57.** $\csc 2u = \frac{1}{\sin 2u} = \frac{1}{2\sin u\cos u} = \frac{1}{2}\cdot\frac{1}{\sin u}\cdot\frac{1}{\cos u} = \frac{1}{2}\csc u \sec u$

59. $\sec 2u = \frac{1}{\cos 2u} = \frac{1}{1 - 2\sin^2 u} = \left(\frac{1}{1 - 2\sin^2 u}\right)\left(\frac{\csc^2 u}{\csc^2 u}\right) = \frac{\csc^2 u}{\csc^2 u - 2}$

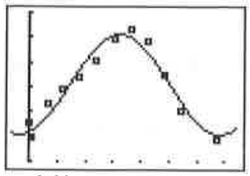
61. $\sec 2u = \frac{1}{\cos 2u} = \frac{1}{\cos^2 u - \sin^2 u} = \left(\frac{1}{\cos^2 u - \sin^2 u}\right)\left(\frac{\sec^2 u\csc^2 u}{\sec^2 u\csc^2 u}\right) = \frac{\sec^2 u\csc^2 u}{\csc^2 u - \sec^2 u}$

63. **(a)**



[–30, 370] by [20, 140]

(b)



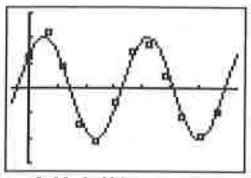
[–30, 370] by [20, 140]

$y = 40.458 \sin(0.0175x - 1.259) + 81.949$

This is a fairly good fit, but not really as good as one might expect from data generated by a sinusoidal physical model.

(c) The residual list: $\{6.34, 11.11, 4.50, -7.12, -10.50, -2.80, 7.07, 8.70, 2.59, -5.60, -9.49, -4.76\}$

(d)



[–30, 370] by [–15, 15]

$y = 10.152 \sin(0.034x + 0.709) - 0.0048$

This is another fairly good fit, which indicates that the residuals are not due to chance. There is a periodic variation that is most probably due to physical causes.

(e) The first regression indicates that the data are periodic and nearly sinusoidal. The second regression indicates that the *variation* of the data around the predicted values is also periodic and nearly sinusoidal. Periodic variation around periodic models is a predictable consequence of bodies orbiting bodies, but ancient astronomers had a difficult time reconciling the data with their simpler models of the universe.

SECTION 5.5

Exploration 1

1. If $BC \leq AB$, the segment will not reach from point B to the dotted line. On the other hand, if $BC > AB$, then a circle of radius BC will intersect the dotted line in a unique point. (Note that the line only extends to the left of point A.)
3. The second point (C_2) is the reflection of the first point (C_1) on the other side of the altitude.
5. If $BC \geq AB$, then BC can only extend to the right of the altitude, thus determining a unique triangle.

Quick Review 5.5

1. bc/d **3.** ad/b **5.** 13.314 **7.** 17.458° **9.** 224.427°

Exercises 5.5

1. $C = 75°; a \approx 4.5; c \approx 5.1$ **3.** $B = 45°; b \approx 15.8; c \approx 12.8$ **5.** $C = 110°; a \approx 12.9; c \approx 18.8$
7. $C = 77°; a \approx 4.1; c \approx 7.3$ **9.** $B \approx 20.1°; C \approx 127.9°; c \approx 25.3$ **11.** $C \approx 37.2°; A \approx 72.8°; a \approx 14.2$
13. 0 **15.** 2 **17.** 2 **19.** $B_1 \approx 72.7°; C_1 \approx 43.3°; c_1 \approx 12.2; B_2 \approx 107.3°; C_2 \approx 8.7°; c_2 \approx 2.7$
21. $A_1 \approx 78.2°; B_1 \approx 33.8°; b_1 \approx 10.8; A_2 \approx 101.8°; B_2 \approx 10.2°; b_2 \approx 3.4$ **23.** **(a)** $6.691 < b < 10$ **(b)** $b \approx 6.691$ or $b \geq 10$
(c) $b < 6.691$ **25.** **(a)** No: this is an SAS case. **(b)** No: only two pieces of information given
27. no triangle is formed **29.** no triangle is formed **31.** $A = 99°; a \approx 28.3; b \approx 19.1$
33. $A_1 \approx 24.6°; B_1 \approx 80.4°; a_1 \approx 20.7; A_2 \approx 5.4°; B_2 \approx 99.6°; a_2 \approx 4.7$ **35.** Cannot be solved by law of sines (an SAS case).
37. **(a)** 54.6 ft **(b)** 51.9 ft **39.** $\approx$ 24.9 ft **41.** 1.9 ft **43.** $\approx$ 108.9 ft **45.** 36.6 mi to A; 28.9 mi to B
47. True. By the Law of Sines, $\frac{\sin A}{a} = \frac{\sin B}{b}$, which is equivalent to $\frac{\sin A}{\sin B} = \frac{a}{b}$. **49.** (c) **51.** (a)
53. **(b)** Possible answers: $a = 1, b = \sqrt{3}, c = 2$ (or any set of three numbers proportional to these) **(c)** Any set of three identical numbers. **55.** **(a)** $h = AB \sin A$ **(b)** $BC < AB \sin A$ **(c)** $BC \geq AB$ or $BC = AB \sin A$ **(d)** $AB \sin A < BC < AB$
57. $AC \approx 8.7$ mi; $BC \approx 12.2$ mi; $h \approx 5.2$ mi

SECTION 5.6

Exploration 1

1. 8475.742818 paces^2 **3.** 0.0014714831 square miles **5.** The estimate of "a little over an acre" seems questionable, but the roughness of their measurement system does not provide firm evidence that it is incorrect. If Jim and Barbara wish to make an issue of it with the owner, they would be well-advised to get some more reliable data.

Quick Review 5.6

1. $A \approx 53.130°$ **3.** $A \approx 132.844°$ **5. (a)** $\cos A = \frac{x^2 + y^2 - 81}{2xy}$ **(b)** $A = \cos^{-1}\left(\frac{x^2 + y^2 - 81}{2xy}\right)$

7. One answer: $(x - 1)(x - 2) = x^2 - 3x + 2$. **9.** One answer: $(x - i)(x + i) = x^2 + 1$

Exercises 5.6

1. $A \approx 30.7°$; $C \approx 18.3°$; $b \approx 19.2$ **3.** $A \approx 76.8°$; $B \approx 43.2°$; $C \approx 60°$ **5.** $B \approx 89.3°$; $C \approx 35.7°$; $a \approx 9.8$ **7.** $A \approx 28.5°$; $B \approx 56.5°$; $c \approx 25.1$ **9.** No triangles possible **11.** $A \approx 24.6°$; $B \approx 99.2°$; $C \approx 56.2°$ **13.** $c_1 \approx 9.487$, $B_1 \approx 72.9°$, $C_1 \approx 65.1°$; $c_2 \approx 5.376$, $B_2 \approx 107.1°$, $C_2 \approx 30.9°$. **15.** No triangles possible **17.** ≈ 222.33 ft^2 **19.** ≈ 107.98 cm^2 **21.** ≈ 8.18 **23.** No triangle is formed **25.** ≈ 216.15 **27.** ≈ 314.05 **29.** ≈ 1.445 radians **31.** ≈ 374.1 square inches **33.** ≈ 498.8 square inches **35.** ≈ 130.42 ft **37. (a)** ≈ 42.5 ft **(b)** The home-to-second segment is the hypotenuse of a right triangle, so the distance from the pitcher's rubber to second base is $60\sqrt{2} - 40 \approx 44.9$ ft. **(c)** $\approx 93.3°$ **39. (a)** $\tan^{-1}\left(\frac{1}{3}\right) \approx 18.4°$ **(b)** ≈ 4.5 ft **(c)** ≈ 7.6 ft **41.** ≈ 12.5 yd **43.** $\cos^{-1}\left(\frac{9}{\sqrt{130}}\right) \approx 37.9°$

45. True. By the Law of Cosines, $b^2 + c^2 - 2bc \cos A = a^2$, which is a positive number. Since $b^2 + c^2 - 2bc \cos A > 0$, it follows that $b^2 + c^2 > 2bc \cos A$. **47.** (b) **49.** (c) **51.** Area $= \frac{nr^2}{2} \sin \frac{360°}{n}$

53. (a) Ship A: $\frac{30.2 - 15.1}{1 \text{ hr}} = 15.1$ knots; Ship B: $\frac{37.2 - 12.4}{2 \text{ hrs}} = 12.4$ knots **(b)** 35.18° **(c)** 34.8 nautical miles **55.** 6.9 in.^{2}

CHAPTER 5 REVIEW EXERCISES

1. $\sin 200°$ **3.** 1 **5.** $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x = (\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x = \cos^3 x - 3 \sin^2 x \cos x = \cos^3 x - 3(1 - \cos^2 x)\cos x = \cos^3 x - 3 \cos x + 3 \cos^3 x = 4 \cos^3 x - 3 \cos x$

7. $\tan^2 x - \sin^2 x = \sin^2 x \left(\frac{1 - \cos^2 x}{\cos^2 x}\right) = \sin^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$

9. $\csc x - \cos x \cot x = \frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x} = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$

11. Recall that $\tan \theta \cot \theta = 1$. $\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 + \cot \theta}{1 - \cot \theta} = \frac{(1 + \tan \theta)(1 - \cot \theta) + (1 + \cot \theta)(1 - \tan \theta)}{(1 - \tan \theta)(1 - \cot \theta)}$
$= \frac{(1 + \tan \theta - \cot \theta - 1) + (1 + \cot \theta - \tan \theta - 1)}{(1 - \tan \theta)(1 - \cot \theta)} = \frac{0}{(1 - \tan \theta)(1 - \cot \theta)} = 0$

13. $\cos^2 \frac{t}{2} = \left[\pm\sqrt{\frac{1}{2}(1 + \cos t)}\right]^2 = \frac{1}{2}(1 + \cos t) = \left(\frac{1 + \cos t}{2}\right)\left(\frac{\sec t}{\sec t}\right) = \frac{\sec t + 1}{2 \sec t}$

15. $\frac{\cos \phi}{1 - \tan \phi} + \frac{\sin \phi}{1 - \cot \phi} = \left(\frac{\cos \phi}{1 - \tan \phi}\right)\left(\frac{\cos \phi}{\cos \phi}\right) + \left(\frac{\sin \phi}{1 - \cot \phi}\right)\left(\frac{\sin \phi}{\sin \phi}\right) = \frac{\cos^2 \phi}{\cos \phi - \sin \phi} + \frac{\sin^2 \phi}{\sin \phi - \cos \phi}$
$= \frac{\cos^2 \phi - \sin^2 \phi}{\cos \phi - \sin \phi} = \cos \phi + \sin \phi$

17. $\sqrt{\frac{1 - \cos y}{1 + \cos y}} = \sqrt{\frac{(1 - \cos y)^2}{(1 + \cos y)(1 - \cos y)}} = \sqrt{\frac{(1 - \cos y)^2}{1 - \cos^2 y}} = \sqrt{\frac{(1 - \cos y)^2}{\sin^2 y}} = \frac{|1 - \cos y|}{|\sin y|} = \frac{1 - \cos y}{|\sin y|}$
—since $1 - \cos y \geq 0$, we can drop that absolute value.

19. $\tan\left(u + \frac{3\pi}{4}\right) = \frac{\tan u + \tan (3\pi/4)}{1 - \tan u \tan (3\pi/4)} = \frac{\tan u + (-1)}{1 - \tan u (-1)} = \frac{\tan u - 1}{1 + \tan u}$ **21.** $\tan \frac{1}{2}\beta = \frac{1 - \cos \beta}{\sin \beta} = \frac{1}{\sin \beta} - \frac{\cos \beta}{\sin \beta} = \csc \beta - \cot \beta$

23. Yes: $\sec x - \sin x \tan x = \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$.

25. Many answers are possible, for example, $(\cos x - \sin x)(1 + 4 \sin x \cos x)$.

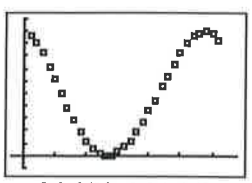
27. Many answers are possible, for example, $1 - 4 \sin^2 x \cos^2 x - 2 \sin x \cos x$.

29. $\frac{\pi}{12} + n\pi$ or $\frac{5\pi}{12} + n\pi$, $n = 0, \pm1, \pm2, \ldots$ **31.** $\frac{-\pi}{4} + n\pi$ **33.** $\tan 1$ **35.** $x \approx 1.12$ **37.** $x \approx 1.15$

39. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **41.** $\left\{\frac{3\pi}{2}\right\}$ **43.** No solutions **45.** $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$ **47.** $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$
49. $y \approx 5\sin(3x + 0.9273)$ **51.** $C = 68°$; $b \approx 3.9$; $c \approx 6.6$ **53.** No triangle is formed **55.** $C = 72°$; $a \approx 2.9$; $b \approx 5.1$
57. $A \approx 44.4°$; $B \approx 78.5°$; $C \approx 57.1°$ **59.** ≈ 7.5 **61.** **(a)** $\approx 5.6 < b < 12$ **(b)** $b \approx 5.6$ or $b \geq 12$ **(c)** $b < 5.6$
63. ≈ 0.6 mi **65.** 1.25 rad **67.** **(a)** $\sin\theta + \frac{1}{2}\sin 2\theta$ **(b)** $\theta = \frac{\pi}{3}; \frac{3}{4}\sqrt{3} \approx 1.30$ square units
69. **(a)** $h = 4000 \sec\frac{\theta}{2} - 4000$ miles **(b)** $\approx 35.51°$ **71.** area of circle $= 256\pi$ cm^2, area outside hexagon $= 256\pi^2 - 384\sqrt{3}$ ≈ 139.140 cm^2 **73.** $\frac{405\pi}{24} \approx 53.01$ cm^3
75. **(a)** By the product-to-sum formula in 74c, $2\sin\frac{u+v}{2}\cos\frac{u-v}{2} = 2 \cdot \frac{1}{2}\left(\sin\frac{u+v+u-v}{2} + \sin\frac{u+v-(u-v)}{2}\right) = \sin u + \sin v$
(b) By the product-to-sum formula in 74c, $2\sin\frac{u-v}{2}\cos\frac{u+v}{2} = 2 \cdot \frac{1}{2}\left(\sin\frac{u-v+u+v}{2} + \sin\frac{u+v-(u+v)}{2}\right)$ $= \sin u + \sin(-v) = \sin u - \sin v$ **(c)** By the product-to-sum formula in 74b, $2\cos\frac{u+v}{2}\cos\frac{u-v}{2}$ $= 2 \cdot \frac{1}{2}\left(\cos\frac{u+v-(u-v)}{2} + \cos\frac{u+v+u-v}{2}\right) = \cos v + \cos u = \cos u + \cos v$
(d) By the product-to-sum formula in 74a, $-2\sin\frac{u+v}{2}\sin\frac{u-v}{2} = -2 \cdot \frac{1}{2}\left(\cos\frac{u+v-(u-v)}{2} - \cos\frac{u+v+u-v}{2}\right)$ $= -(\cos v - \cos u) = \cos u - \cos v$
77. **(a)** Any inscribed angle that intercepts an arc of 180° is a right angle. **(b)** Two inscribed angles that intercept the same arc are congruent. **(c)** In right $\triangle A'BC$, $\sin A' = \frac{\text{opp}}{\text{hyp}} = \frac{a}{d}$. **(d)** Because $\angle A'$ and $\angle A$ are congruent, $\frac{\sin A}{a} = \frac{\sin A'}{a} = \frac{a/d}{a} = \frac{1}{d}$.
(e) Of course. They both equal $\frac{\sin A}{a}$ by the Law of Sines.

Chapter 5 Project

1.



[−2, 34] by [−0.1, 1.1]

5. One possible model is $y \approx 0.5\sin\left(\frac{2\pi}{30.5}(x - 21.375)\right) + 0.5$.

SECTION 6.1

Quick Review 6.1

1. $\frac{9\sqrt{3}}{2}$, 4.5 **3.** −5.36, −4.50 **5.** 33.85° **7.** 60.95° **9.** 248.20°

Exercises 6.1

5. $\langle 5, 2\rangle, \sqrt{29}$ **7.** $\langle -5, 1\rangle, \sqrt{26}$ **9.** $\langle -2, -24\rangle, 2\sqrt{145}$ **11.** $\langle -11, -7\rangle, \sqrt{170}$ **13.** $\langle 1, 7\rangle$ **15.** $\langle -3, 8\rangle$
17. $\langle 4, -9\rangle$ **19.** $\langle -4, -18\rangle$ **21.** $\approx -0.45\mathbf{i} + 0.89\mathbf{j}$ **23.** $\approx -0.45\mathbf{i} - 0.89\mathbf{j}$ **25.** **(a)** $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle$ **(b)** $\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$
27. **(a)** $\left\langle -\frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}}\right\rangle$ **(b)** $-\frac{4}{\sqrt{41}}\mathbf{i} - \frac{5}{\sqrt{41}}\mathbf{j}$ **29.** $\approx \langle 16.31, 7.61\rangle$ **31.** $\approx \langle -14.52, 44.70\rangle$ **33.** 5; $\approx 53.13°$
35. 5; $\approx 306.87°$ **37.** 7; 135° **39.** $\langle \sqrt{2}, -\sqrt{2}\rangle$ **41.** $\approx \langle -223.99, 480.34\rangle$ **43.** **(a)** $\approx \langle -111.16, 305.40\rangle$
(b) 362.84 mph; 337.84° **45.** **(a)** $\approx \langle 3.42, 9.40\rangle$ **47.** $\approx \langle 2.20, 1.43\rangle$ **49.** $|\mathbf{F}| \approx 100.33$ lb and $\theta \approx -1.22°$
51. $\approx 342.86°$; ≈ 9.6 mph **53.** about 13.66 mph and about 7.07 mph **55.** True. **u** and $-\mathbf{u}$ have the same length but opposite directions. Thus, the length of $-\mathbf{u}$ is also 1. **57.** (d) **59.** (a)