SECTION 9.1

Exploration 1

1. 6 **3.** No

Quick Review 9.1

1. 52 **3.** 6 **5.** 10 **7.** 11 **9.** 64

Exercises 9.1

1. 6 **3.** 120 **5.** 12 **7.** 362,880 (ALGORITHM) **9.** 34,650 **11.** 1716 **13.** 24 **15.** 30 **17.** 120 **19.** combinations **21.** combinations **23.** 19,656,000 **25.** 36 **27.** 2300 **29.** 17,296 **31.** 37,353,738,800 **33.** 41 **35.** 7776 **37.** 511 **39.** 12 **41.** 1024 **43.** True. Both equal $\frac{n!}{a!b!}$. **45.** (d) **47.** (b)

51. (a) 12 **(b)** There are 12 factors of 5 in 50!, one in each of 5, 10, 15, 20, 30, 35, 40, and 45 and two in each of 25 and 50. Each factor of 5, when paired with one of the 47 factors of 2, yields a factor of 10 and consequently a 0 at the end of 50!
55. 3 **57.** $\approx$ 20,136 years

SECTION 9.2

Exploration 1

1. 1, 3, 3, 1; These are (in order) the coefficients in the expansion of $(a+b)^3$. **3.** {1 5 10 10 5 1}; These are (in order) the coefficients in the expansion of $(a+b)^5$.

Quick Review 9.2

1. $x^2 + 2xy + y^2$ **3.** $25x^2 - 10xy + y^2$ **5.** $9s^2 - 12st + 4t^2$ **7.** $u^3 + 3u^2v + 3uv^2 + v^3$
9. $8x^3 - 36x^2y + 54xy^2 - 27y^3$

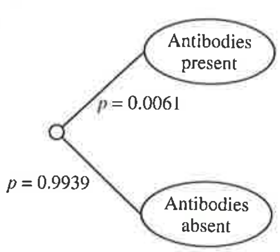
Exercises 9.2

1. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ **3.** $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
5. $x^3 + 3x^2y + 3xy^2 + y^3$ **7.** $p^8 + 8p^7q + 28p^6q^2 + 56p^5q^3 + 70p^4q^4 + 56p^3q^5 + 28p^2q^6 + 8pq^7 + q^8$
9. 36 **11.** 1 **13.** 364 **15.** 126,720 **17.** $f(x) = x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$
19. $h(x) = 128x^7 - 448x^6 + 672x^5 - 560x^4 + 280x^3 - 84x^2 + 14x - 1$ **21.** $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
23. $x^3 - 6x^{5/2}y^{1/2} + 15x^2y - 20x^{3/2}y^{3/2} + 15xy^2 - 6x^{1/2}y^{5/2} + y^3$ **25.** $x^{-10} + 15x^{-8} + 90x^{-6} + 270x^{-4} + 405x^{-2} + 243$
35. True. The signs of the coefficients are determined by the powers of the $(-y)$. **37.** (c) **39.** (a)
41. (a) 1, 3, 6, 10, 15, 21, 28, 36, 45, 55 **(b)** They appear diagonally down the triangle, starting with either of the 1's in row 2. **43.** $2^n = (1+1)^n = \binom{n}{0}1^n1^0 + \binom{n}{1}1^{n-1}1^1 + \binom{n}{2}1^{n-2}1^2 + \cdots + \binom{n}{n}1^01^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

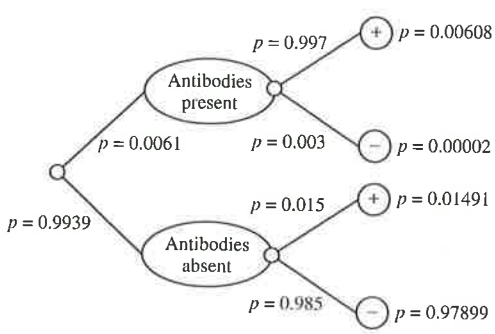
SECTION 9.3

Exploration 1

1.



3.



5. $\approx$ 0.29

Quick Review 9.3

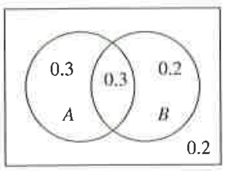
1. 2 **3.** 8 **5.** 2,598,960 **7.** 120 **9.** $\frac{1}{12}$

Exercises 9.3

1. $\frac{1}{9}$ **3.** $\frac{5}{12}$ **5.** $\frac{1}{4}$ **7.** $\frac{5}{12}$ **9. (a)** No; the numbers do not add up to 1. **(b)** Yes; assuming the gerbil cannot be in more than one compartment at a time. **11.** 0.4 **13.** 0.2 **15.** 0.7 **17.** 0.09

19. 0.08 **21.** 0.64 **23.** $\frac{1}{134{,}596}$ **25.** $\frac{5}{3542}$

27. (a)



(b) 0.3 **(c)** 0.2 **(d)** 0.2 **(e)** Yes

29. 0.64 **31.** $\frac{3}{5}$ **33.** $\frac{19}{30}$ **35. (a)** 0.67 **(b)** 0.33 **39. (a)** $\frac{86}{127}$ **(b)** $\frac{91}{127}$ **(c)** $\frac{62}{127}$

41. $\frac{1}{36}$ **43.** $\frac{1}{1024}$ **45.** $\frac{1}{1024}$ **47.** $\frac{45}{1024}$ **49.** $\frac{1023}{1024}$

51. False. A sample space consists of outcomes, which are not necessarily equally likely. **53.** (d) **55.** (a)

57. (a)

Type of Bagel	Probability
Plain	0.37
Onion	0.12
Rye	0.11
Cinnamon Raisin	0.25
Sourdough	0.15

(b) ≈ 0.051

59. (a) ≈ 2 **(b)** Yes **(c)** $\approx 1.913\%$

61. (a) \$1.50 **(b)** $\frac{1}{3}$

SECTION 9.4

Exploration 1

1. 45 **3.** 1 **5.** $\frac{1}{3}$

Exploration 2

1. $1 + 2 + 3 + \cdots + 99 + 100$ **3.** 101 **5.** The sum in 4 involves two copies of the same progression, so it doubles the sum of the progression. The answer is 5050.

Quick Review 9.4

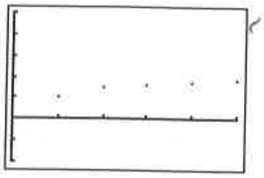
1. 19 **3.** 80 **5.** $\frac{10}{11}$ **7.** 2560 **9.** 15

Exercises 9.4

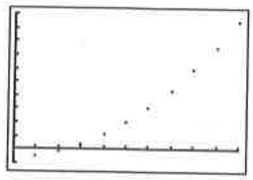
1. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}; \frac{101}{100}$ **3.** 0, 6, 24, 60, 120, 210; 999,900 **5.** 8, 4, 0, −4; −20 **7.** 2, 6, 18, 54; 4374 **9.** 2, −1, 1, 0; 3

11. (a) 4 **(b)** 42 **(c)** $a_1 = 6$ and $a_n = a_{n-1} + 4$ for $n \geq 2$ **(d)** $a_n = 6 + 4(n - 1)$ **13. (a)** 3 **(b)** 22 **(c)** $a_1 = -5$ and $a_n = a_{n-1} + 3$ for $n \geq 2$ **(d)** $a_n = -5 + 3(n - 1)$ **15. (a)** 3 **(b)** 4374 **(c)** $a_1 = 2$ and $a_n = 3a_{n-1}$ for $n \geq 2$ **(d)** $a_n = 2 \cdot 3^{n-1}$ **17. (a)** −2 **(b)** −128 **(c)** $a_1 = 1$ and $a_n = -2a_{n-1}$ for $n \geq 2$ **(d)** $a_n = (-2)^{n-1}$

19. $a_1 = -20$ and $a_n = a_{n-1} + 4$ for $n \geq 2$ **21.** $a_1 = \pm\frac{3}{2}$, $r = \pm 2$ and $a_n = 3(\pm 2)^{n-2}$

23. 

[0, 5] by [−2, 5]

25. 

[0, 10] by [−10, 100]

27. $\sum_{k=1}^{11} (6k - 13)$ **29.** $\sum_{k=1}^{n+1} k^2$ **31.** $\sum_{k=0}^{\infty} 6(-2)^k$

33. 18 **35.** 3240 **37.** 975 **39.** 24,573 **41.** $50.4(1 - 6^{-9}) \approx 50.4$ **43.** 155 **45.** $\frac{8}{3}(1 - 2^{-12}) \approx 2.666$
47. −196,495,641 **49. (a)** 0.3, 0.33, 0.333, 0.3333, 0.33333, 0.333333; convergent **(b)** 1, −1, 2, −2, 3, −3; divergent
51. Yes; 12 **53.** No **55.** Yes; 1 **57.** $\frac{707}{99}$ **59.** $-\frac{17{,}251}{999}$ **61.** 700, 702.3, 704.6, 706.9, ... , 815, 817.3
63. (a) 1.1 **(b)** $20{,}000(1.1)^n$ **(c)** \$370,623.34 **65.** 120; $1 + \frac{0.07}{12}$ **(b)** \$20,770.18 **67.** 775 **69.** ≈ 24.05 m
71. True. The common ratio r must be positive, so the sign of the first term determines the sign of every number in the sequence.
73. (a) **75.** (d) **77. (b)** 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 **79. (b)** $a_n \to 2\pi$ as $n \to \infty$ **83. (a)** Heartland: 19,237,759 people; Southeast: 42,614,977 people **(b)** Heartland: 517,825 mi²; Southeast: 348,999 mi² **(c)** Heartland: ≈ 37.15 people/mi²; Southeast: ≈ 122.11 people/mi² **85.** $a_1 = [1\ 1]$, $a_2 = [1\ 2]$, $a_3 = [2\ 3]$, $a_4 = [3\ 5]$, $a_5 = [5\ 8]$, $a_6 = [8\ 13]$, $a_7 = [13\ 21]$. The entries in the terms of this sequence are successive pairs of terms from the Fibonacci sequence. **87.** $S_n = F_{n+2} - 1$

SECTION 9.5

Exploration 1

Start with the rightmost peg if n is odd and the middle peg if n is even.

Exploration 2

1. Yes **3.** All prime

Quick Review 9.5

1. $n^2 + 5n$ **3.** $k^3 + 3k^2 + 2k$ **5.** $(k + 1)^3$ **7.** 5; $t + 4$; $t + 5$ **9.** $\frac{1}{2}; \frac{2k}{3k+1}; \frac{2k+2}{3k+4}$

Exercises 9.5

1. P_n: $2 + 4 + 6 + \cdots + 2n = n^2 + n$. P_1 is true: $2(1) = 1^2 + 1$. Now assume P_k is true: $2 + 4 + 6 + \cdots + 2k = k^2 + k$. Add $2(k + 1)$ to both sides: $2 + 4 + 6 + \cdots + 2k + 2(k + 1) = k^2 + k + 2(k + 1) = k^2 + 3k + 2 = k^2 + 2k + 1 + k + 1 = (k + 1)^2 + (k + 1)$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.
3. P_n: $6 + 10 + 14 + \cdots + (4n + 2) = n(2n + 4)$. P_1 is true: $4(1) + 2 = 1(2(1) + 4)$. Now assume P_k is true: $6 + 10 + 14 + \cdots + (4k + 2) = k(2k + 4)$. Add $4(k + 1) + 2 = 4k + 6$ to both sides: $6 + 10 + 14 + \cdots + (4k + 2) + [4(k + 1) + 2] = k(2k + 4) + 4k + 6 = 2k^2 + 8k + 6 = (k + 1)(2k + 6) = (k + 1)[2(k + 1) + 4]$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.
5. P_n: $5n - 2$. P_1 is true: $a_1 = 5 \cdot 1 - 2 = 3$. Now assume P_k is true: $a_k = 5k - 2$. To get a_{k+1}, add 5 to a_k; that is, $a_{k+1} = (5k - 2) + 5 = 5(k + 1) - 2$. This shows that P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.
7. P_n: $a_n = 2 \cdot 3^{n-1}$. P_1 is true: $a_1 = 2 \cdot 3^{1-1} = 2 \cdot 3^0 = 2$. Now assume P_k is true: $a_k = 2 \cdot 3^{k-1}$. To get a_{k+1}, multiply a_k by 3; that is, $a_{k+1} = 3 \cdot 2 \cdot 3^{k-1} = 2 \cdot 3k = 2 \cdot 3^{(k+1)-1}$. This shows that P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.
9. P_1: $1 = \frac{1(1+1)}{2}$. P_k: $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$. P_{k+1}: $1 + 2 + \cdots + k + (k + 1) = \frac{(k+1)(k+2)}{2}$.
11. P: $\frac{1}{1 \cdot 2} = \frac{1}{1+1}$. P_k: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$. P_{k+1}: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$.
13. P_n: $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$. P_1 is true: $4(1) - 3 = 1 \cdot (2 \cdot 1 - 1)$. Now assume P_k is true: $1 + 5 + 9 + \cdots + (4k - 3) = k(2k - 1)$. Add $4(k + 1) - 3 = 4k + 1$ to both sides: $1 + 5 + 9 + \cdots + (4k - 3) + [4(k + 1) - 3] = k(2k - 1) + 4k + 1 = 2k^2 + 3k + 1 = (k + 1)(2k + 1) = (k + 1)[2(k + 1) - 1]$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.
15. P_n: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$. P_1 is true: $\frac{1}{1 \cdot 2} = \frac{1}{1+1}$. Now assume P_k is true: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)}$

$= \frac{k}{k+1}$. Add $\frac{1}{(k+1)(k+2)}$ to both sides: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{k+1}{k+2} = \frac{k+1}{k+1+1}$, so P_{k+1} *is* true. Therefore, P_n is true for all $n \geq 1$.

17. P_n: $2^n \geq 2n$. P_1 is true: $2^1 \geq 2 \cdot 1$ (in fact, they are equal). Now assume P_k is true: $2^k \geq 2k$. Then $2^{k+1} = 2 \cdot 2^k \geq 2 \cdot 2k = 2 \cdot (k + k) \geq 2(k+1)$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.

19. P_n: 3 is a factor of $n^3 + 2n$. P_1 is true: 3 is a factor of $1^3 + 2 \cdot 1 = 3$. Now assume P_k is true: 3 is a factor of $k^3 + 2k$. Then $(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + (2k + 2) = (k^3 + 2k) + 3(k^2 + k + 1)$. Since 3 is a factor of both terms, it is a factor of the sum, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.

21. P_n: The sum of the first n terms of a geometric sequence with first term a_1 and common ratio $r \neq 1$ is $\frac{a_1(1-r^n)}{1-r}$. P_1 is true: $a_1 = \frac{a_1(1-r^1)}{1-r}$. Now assume P_k is true so that $a_1 + a_1 r + \cdots + a_1 r^{k-1} = \frac{a_1(1-r^k)}{(1-r)}$. Add $a_1 r^k$ to both sides: $a_1 + a_1 r + \ldots + a_1 r^{k-1} + a_1 r^k = \frac{a_1(1-r^k)}{(1-r)} + a_1 r^k = \frac{a_1(1-r^k) + a_1 r^k(1-r)}{1-r} = \frac{a_1 - a_1 r^k + a_1 r^k - a_1 r^{k+1}}{1-r} = \frac{a_1 - a_1 r^{k+1}}{1-r}$, so P_{k+1} is true. Therefore, P_n is true for all positive integers n.

23. P_n: $\sum_{k=1}^{n} = \frac{n(n+1)}{2}$. P_1 is true: $\sum_{k=1}^{1} k = 1 = \frac{1 \cdot 2}{2}$. Now assume P_k is true: $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$. Add $(k+1)$ to both sides, and we have $\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2}$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$. **25.** 125,250 **27.** $\frac{(n-3)(n+4)}{2}$ **29.** $\approx 3.44 \times 10^{10}$ **31.** $\frac{n(n^2 - 3n + 8)}{3}$

33. $\frac{n(n-1)(n^2 + 3n + 4)}{4}$

35. The inductive step does not work for 2 people. Sending them alternately out of the room leaves 1 person (and one blood type) each time, but we cannot conclude that their blood types will match *each other.*

37. False. Mathematical induction is used to show that a statement P_n is true for all positive integers. **39.** (e) **41.** (b)

43. P_n: 2 is a factor of $(n+1)(n+2)$. P_1 is true because 2 is a factor of (2)(3). Now assume P_k is true so that 2 is a factor of $(k+1)(k+2)$. Then $[(k+1)+1][(k+2)+2] = (k+2)(k+3) = k^2 + 5k + 6 = k^2 + 3k + 2 + 2k + 4 = (k+1)(k+2) + 2(k+2)$. Since 2 is a factor of both terms of this sum, it is a factor of the sum, and so P_{k+1} is true. Therefore, P_n is true for all positive integers n.

45. Given any two consecutive integers, one of them must be even. Therefore, their product is even. Since $n + 1$ and $n + 2$ are consecutive integers, their product is even. Therefore, 2 is a factor of $(n+1)(n+2)$.

47. P_n: $F_{n+2} - 1 = \sum_{k=1}^{n} F_k$. P_1 is true since $F_{1+2} - 1 = F_3 - 1 = 2 - 1 = 1$, which equals $\sum_{k=1}^{1} F_k = 1$. Now assume that P_k is true: $F_{k+2} - 1 = \sum_{i=1}^{k} F_i$. Then $F_{(k+1)+2} - 1 = F_{k+3} - 1 = F_{k+1} + F_{k+2} - 1 = (F_{k+2} - 1) + F_{k+1} = \left(\sum_{i=1}^{k} F_i\right) + F_{k+1} = \sum_{i=1}^{k+1} F_i$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.

49. P_n: $a - 1$ is a factor of $a^n - 1$. P_1 is true because $a - 1$ is a factor of $a - 1$. Now assume P_k is true so that $a - 1$ is a factor of $a^k - 1$. Then $a^{k+1} - 1 = a \cdot a^k - 1 = a(a^k - 1) + (a - 1)$. Since $a - 1$ is a factor of both terms in the sum, it is a factor of the sum, and so P_{k+1} is true. Therefore, P_n is true for all positive integers n.

51. P_n: $3n - 4 \geq n$ for $n \geq 2$. P_2 is true since $3 \cdot 2 - 4 \geq 2$. Now assume that P_k is true: $3k - 4 \geq k$. Then $3(k + 1) - 4 = 3k + 3 - 4 = (3k - 4) + 3 \geq k + 3 \geq k + 1$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 2$.

53. Use P_3 as the anchor and obtain the inductive step by representing any n-gon as the union of a triangle and an $(n - 1)$-gon.

SECTION 9.6

Exploration 1

1. The average is about 12.8. **3.** Alaska, Colorado, Georgia, Texas, and Utah

Quick Review

1. $\approx 15.48\%$ **3.** $\approx 14.44\%$ **5.** ≈ 1723 **7.** $235 thousand **9.** 1 million

Exercises

1.

0	5 8 9
1	3 4 6
2	3 6 8
3	3 9
4	
5	
6	1

61 is an outlier.

3.

5	9
6	1 2
6	7 8 9 9
7	0 1 2 2 3

This stemplot shows most of the life expectancies of males to be clustered near 70, with three lower values clustered near 60.

5.

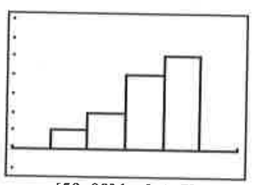
Male		Female
9	5	
2 1	6	
9 9 8 7	6	6 7 8
3 2 2 1 0	7	3 4 4
	7	5 7 7 9 9 9

This stemplot shows two distributions of similar shapes, but the women's life expectancies are uniformly about 5 years higher than the men's.

7.

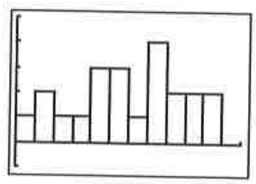
Life expectancy (years)	Frequency
55.0 — 59.9	1
60.0 — 64.9	2
65.0 — 69.9	4
70.0 — 74.9	5

9.



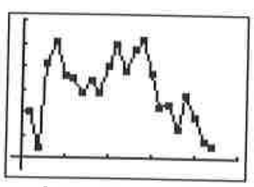
[50, 80] by [−1, 7]

11.



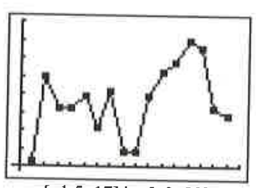
[0, 60] by [−1, 5]

13.



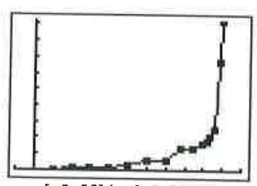
[−1, 25] by [−5, 60]

15.



[−1.5, 17] by [−2, 80]

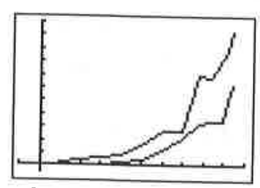
17.



[−5, 55] by [−5, 9200]

Men's winnings seem to be growing exponentially.

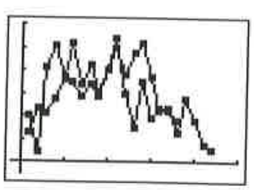
19.



[−5, 50] by [−100, 2200]

Men's winnings are greater than women's with the disparity increasing greatly between 1998 and 2000.

21.



[−1, 25] by [−5, 60]

The two home run hitters enjoyed similar success.

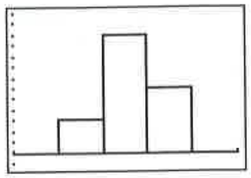
23. **(a)**

Stem	Leaf
28	2
29	3 7
30	
31	6 7
32	7 8
33	5 5 5 8
34	2 8 8
35	3 3 4
36	3 7
37	
38	5

(b)

Interval	Frequency
25.0–29.9	3
30.0–34.9	11
35.0–39.9	6

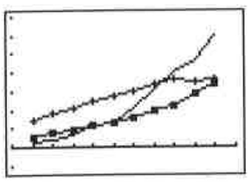
(c)



[20, 45] by [−1, 13]

(d) Time is not a variable in the data.

25.

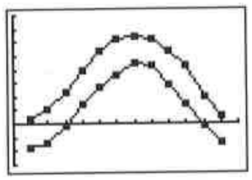


[1890, 2000] by [−5, 35]

— = California, + = New York, ■ = Texas

27. False. The empty branches are important for visualizing the distribution of the data

29. (c) **31.** (a) **35.**



[0, 13] by [−15, 40]

SECTION 9.7

Exploration 1

1. Figure (b)

Quick Review

1. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$ **3.** $\frac{1}{7}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)$ **5.** $\frac{1}{5}[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_5 - \bar{x})^2]$

7. $\sum_{i=1}^{8} x_i f_i$ **9.** $\frac{1}{50} \sum_{i=1}^{50} (x_i - \bar{x})^2$

Exercises

1. **(a)** statistic **(b)** parameter **3.** 26.8 **5.** ≈ 60.12 **7.** 3.9 million **9.** ≈ 10.1 satellites **11.** 2

13. 30 home runs/year; 29.8 home runs/year; Mays **15.** What-Next Fashion

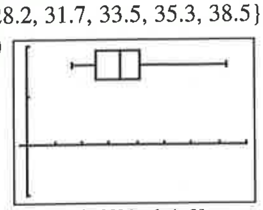
17. median: 87.85; mode: None **21.** **(a)** ≈ 6.42°C **(b)** ≈ 6.49°C **(c)** The weighted average is the better indicator.

23. Mark McGwire:
Five-number summary: {3, 25.5, 36, 50.5, 70}
Range: 67
IQR: 25
No outliers

Barry Bonds:
Five-number summary: {16, 25, 34, 41, 73}
Range: 57
IQR: 16
Outlier: 73

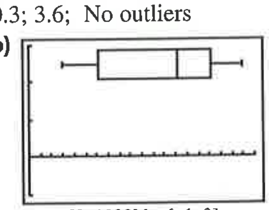
25. {28.2, 31.7, 33.5, 35.3, 38.5}; 10.3; 3.6; No outliers

27. **(a)**



[–3, 80] by [–1, 2]

(b)

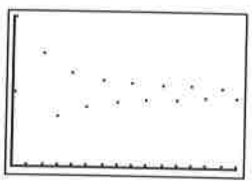


[0, 1000] by [–1, 3]

29. $\frac{3}{11}$ **31.** **(a)** Mays **(b)** Mays **33.** $\sigma \approx 9.08, \sigma^2 = 82.5$ **35.** $\sigma \approx 268.36; \sigma^2 \approx 72{,}017.48$ **37.** $\sigma \approx 1.53, \sigma^2 = 2.34$
39. No **41.** **(a)** 68% **(b)** 2.5% **(c)** statistic **43.** The median is a resistant measure. **45.** (a) **47.** (b)
49. There are many possible answers; examples are given. **(a)** {2, 2, 2, 3, 6, 8, 20} **(b)** {1, 2, 3, 4, 6, 48, 48}
(c) {−20, 1, 1, 1, 2, 3, 4, 5, 6} **51.** No **55.** 71.68 years **57.** 5%

CHAPTER 9 REVIEW EXERCISES

1. 792 **3.** 18,564 **5.** 3,991,680 **7.** 43,670,016 **9.** 14,508,000 **11.** 8,217,822,536 **13.** 26 **15.** 325
17. **(a)** 5040 (Meg Ryan) **(b)** 778,377,600 (Britney Spears) **19.** $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$
21. $243x^{10} + 405x^8y^3 + 270x^6y^6 + 90x^4y^9 + 15x^2y^{12} + y^{15}$
23. $512a^{27} - 2304a^{24}b^2 + 4608a^{21}b^4 - 5376a^{18}b^6 + 4032a^{15}b^8 - 2016a^{12}b^{10} + 672a^9b^{12} - 144a^6b^{14} + 18a^3b^{16} - b^{18}$ **25.** −1320 **27.** {1, 2, 3, 4, 5, 6} **29.** {13, 16, 31, 36, 61, 63} **31.** {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
33. {HHH, TTT} **35.** $\frac{1}{64}$ **37.** $\frac{1}{4}$ **39.** 0.25 **41.** 0.24 **43.** 0.64 **45.** **(a)** 0.5 **(b)** 0.15 **(c)** 0.35 **(d)** ≈ 0.43
47. 0, 1, 2, 3, 4, 5; 39 **49.** −1, 2, 5, 8, 11, 14; 32 **51.** −5, −3.5, −2, −0.5, 1, 2.5; 11.5 **53.** −3, 1, −2, −1, −3, −4; −76
55. Arithmetic with $d = -2.5$; $a_n = 14.5 - 2.5n$ **57.** Geometric with $r = 1.2$; $a_n = 10 \cdot (1.2)^{n-1}$ **59.** Arithmetic with $d = 4.5$; $a_n = 4.5n - 15.5$ **61.** $a_n = 3(-4)^{n-1}$; $r = -4$ **63.** −4 **65.** −985.5 **67.** 21/8 **69.** 59,048
71. $3280.\overline{4}$ **73.**



[0, 15] by [0, 2]

75. $27,441.91 **77.** Converges; 6 **79.** Diverges **81.** Converges; 3
83. $\sum_{k=1}^{21} (5k - 13)$ **85.** $\sum_{k=0}^{\infty} (2k+1)^2$ or $\sum_{k=1}^{\infty} (2k-1)^2$ **87.** $\frac{n(3n+5)}{2}$ **89.** 4650

91. $P_n: 1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$. P_1 is true: $\frac{1(1+1)}{2} = \frac{1(1+1)(1+2)}{6}$.
Now assume P_k is true: $1 + 3 + 6 + \cdots + \frac{k(k+1)}{2} = \frac{k(k+1)(k+2)}{6}$.
Add $\frac{(k+1)(k+2)}{2}$ to both sides: $1 + 3 + 6 + \cdots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2} = \frac{k(k+1)(k+2)}{6} + \frac{(k+1)(k+2)}{2}$
$= (k+1)(k+2)\left(\frac{k}{6} + \frac{1}{2}\right) = (k+1)(k+2)\left(\frac{k+3}{6}\right) = \frac{(k+1)((k+1)+1)((k+1)+2)}{6}$, so P_{k+1} is true.
Therefore, P_n is true for all $n \geq 1$.

93. P_n: $2^{n-1} \leq n!$. P_1 is true: its says that $2^{1-1} \leq 1!$ (they are equal). Now assume P_k is true: $2^{k-1} \leq k!$. Then $2^{(k+1)-1} = 2 \cdot 2^{k-1} \leq 2 \cdot k! \leq (k+1)k! = (k+1)!$, so P_{k+1} is true. Therefore, P_n is true for all $n \geq 1$.

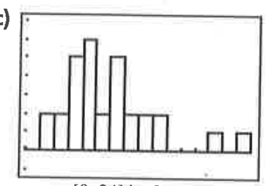
95. **(a)**

9	1 2
10	6 7
11	4 5 5 7 7
12	0 2 4 6 7 7
13	5 6
14	1 6 7 7 8
15	4 8
16	1 4
17	0 6
18	
19	
20	
21	9
22	
23	4

b)

Price	Frequency
90,000– 99,999	2
100,000–109,999	2
110,000–119,999	5
120,000–129,999	6
130,000–139,999	2
140,000–149,999	5
150,000–159,999	2
160,000–169,999	2
170,000–179,999	2
210,000–219,999	1
230,000–239,999	1

(c)



[8, 24] by [−1, 7]

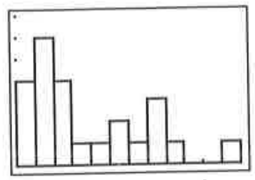
97. (a)

Stem	Leaf
12	0 0 4 4
13	1 1 2 6 7 9
14	0 3 4 8
15	6
16	3
17	7 9
18	0
19	0 1 7
20	2
21	
22	
23	0

b)

Length (in seconds)	Frequency
120–129	4
130–139	6
140–149	4
150–159	1
160–169	1
170–179	2
180–189	1
190–199	3
200–209	1
210–219	0
220–229	0
230–239	1

(c)

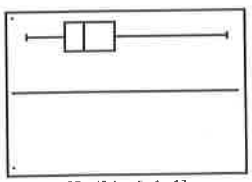


[120, 240] by [0, 7]

99. Five-number summary: {9.1, 11.7, 13.1, 15.4, 23.4}; Range: 14.3; IQR: 3.7; $\sigma \approx 3.19$, $\sigma^2 \approx 10.14$; Outliers: 21.9 and 23.4

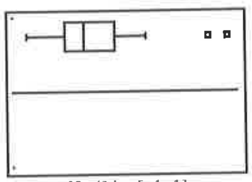
101. Five-number summary: {120, 131.5, 143.5, 179.5, 230}; Range: 110; IQR: 48; $\sigma = 29.9$, $\sigma^2 = 891.4$; No outliers

103. (a)



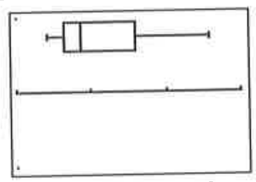
[8, 4] by [−1, 1]

(b)



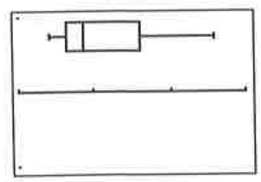
[8, 4] by [−1, 1]

105. (a)



[100, 250] by [−1, 1]

(b)



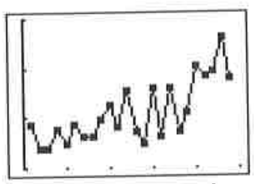
[100, 250] by [−1, 1]

107.

4 0 0	12	4
9 2 1	13	1 6 7
8 4 3 0	14	
	15	6
3	16	
7	17	9
	18	0
	19	0 1 7
	20	2
	21	
	22	
	23	0

The songs released in the earlier years tended to be shorter.

109.



[−1, 25] by [100, 250]

Again, the data demonstrates that songs appearing later tended to be longer.

111. 1 9 36 84 126 126 84 36 9 1

113. (a) ≈ 0.922 **(b)** ≈ 0.075

Chapter 9 Project

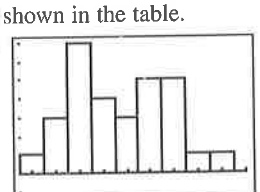
Answers are based on the sample data shown in the table.

1.

Stem	Leaf
5	
5	9
6	1 1 2 3 3 3 4 4 4 4
6	5 6 6 6 7 8 8 9 9 9
7	0 0 1 1 1 2 2 3
7	5

66 or 67 inches

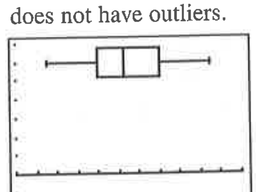
3.



[59, 78] by [−1, 7]

5. The data set is well distributed and probably does not have outliers.

7.



[56, 78] by [−1, 7]