Sequences and Series Review

1. Consider the sequence 1, $\frac{2}{5}$, $\frac{4}{25}$, $\frac{8}{125}$,...
   a. Describe the pattern formed in the sequence.
   b. Find the next three terms.

2. i.) Write a formula for the arithmetic sequence 6, 0, –6, –12, –18, ...
   ii.) Then find the next term. _____

   Describe the pattern in the sequence. Find the next three terms.

3. 800, 320, 128, 51.2,...

4. Evaluate the series $\sum_{n=4}^{6} 5n$.

5. Find a formula for the sequence. *1,5, 9, 13*....

6. Use summation notation to write the series 3 + 5 + 7 + 9 + ... for 9 terms.

   Evaluate the infinite geometric series. Round to the nearest hundredth if necessary.

7. 1 + 0.5 + 0.25 + ...

8. Find the 30th term of the sequence 5, –1, –7, –13, ...

9. Consider the sequence 16, –8, 4, –2, 1, ...
   a. Describe the pattern formed in the sequence.
   b. Find the next three terms.

10. Given the formula $a_n = -3(a_{n-1} - 1)$, find the first five terms of the sequence starting with 5.

11. What is the next term: 2, -6, 18, -54, . . .

12. The sequence 15, 21, 27, 33, 39, ..., 75 has 11 terms. Evaluate the related series.

13. Consider the infinite geometric series $\sum_{n=1}^{\infty} -4\left(\frac{1}{3}\right)^{n-1}$.
    a. Write the first four terms of the series.
    b. Does the series *diverge* or *converge*?
    c. If the series has a sum, find the sum.

14. Consider the sequence 8, 6, 3, –1, –6, ...
    a. Find the next two terms of the sequence.
    b. Write an explicit formula for the sequence.
    c. Write a recursive formula for the sequence.

15. Describe the pattern with a geometric sequence formula. Then find the next two numbers in the pattern. 1, .5, .25, .125, . . .

16. Describe the pattern in the sequence. Find the next three terms.
13, 15, 17, 19, ...

Does the infinite geometric series diverge or converge? Explain.

17. $2 + 10 + 50 + 250 + \ldots$

18. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \ldots$

a. It diverges; it has a sum.
b. It diverges; it does not have a sum.
c. It converges; it has a sum.
d. It converges; it does not have a sum.

19. For the series $\sum_{n=4}^{9} (n + 1)$, find the number of terms in the series.

a. 4 terms
b. 13 terms
c. 6 terms
d. 5 terms

20. Consider the sequence –7, –5.6, –4.2, –2.8, –1.4, ...
a. Write an explicit formula for the sequence. Explain your steps.
b. Write a recursive formula for the sequence. Explain your steps.
c. Suppose you need to find the 50th term of the sequence. Explain which formula you would use.
d. Which term is the number 103.6? Explain your method for solving this problem.

21.

21. Find the first four terms and the tenth term of the sequence whose nth term is defined by $a_n = n^2 - 1$

22. Find the common difference, the fourth term, and the nth term in the arithmetic sequence $80, 76, 72, \ldots$

23. The first term of a geometric sequence is 25, and the fourth term is $\frac{1}{5}$. Find the common ratio, and the fifth term.

24. The first term of an arithmetic sequence is 10, and the tenth term is 2. Find the sum of the first ten terms.

25. Find the common difference and the 100^{th} term of the sequence in (a).

AFM Unit 4 Test - Review
Answer Section

1. ANS:
   a. Add 1 to the numerator and 2 to the denominator of the previous term.
   b. $\frac{6}{11}, \frac{7}{13}, \frac{8}{15}$

   PTS: 1 DIF: L2 REF: 11-1 Mathematical Patterns
   OBJ: 11-1.1 Identifying Mathematical Patterns
   NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 STA: NC A2.1.08
   TOP: 11-1 Example 1 KEY: sequence | pattern | multi-part question
2. ANS:
   $a_n = 6 - 6(n - 1)$; –24

   PTS: 1 DIF: L1 REF: 11-1 Mathematical Patterns
   OBJ: 11-1.2 Using Formulas to Generate Mathematical Patterns
   TOP: 11-1 Example 3 KEY: sequence | pattern | recursive formula
   MSC: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53
3. ANS:
   Divide by 2.5; 16, 6.4, 2.56.

   PTS: 1 DIF: L3 REF: 11-1 Mathematical Patterns
   OBJ: 11-1.1 Identifying Mathematical Patterns
   NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 STA: NC A2.1.08
   TOP: 11-1 Example 1 KEY: sequence | pattern
4. ANS:
   165

   PTS: 1 DIF: L2 REF: 11-4 Arithmetic Series
   OBJ: 11-4.2 Using Summation Notation
   NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP J.1.7 TOP: 11-4 Example 4
   KEY: arithmetic series | summation notation | evaluating a series
5. ANS:
   $a_n = 1 + 4(n - 1)$

   PTS: 1 DIF: L3 REF: 11-1 Mathematical Patterns
   OBJ: 11-1.1 Identifying Mathematical Patterns TOP: 11-1 Example 1
   KEY: sequence | pattern
   MSC: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP |

S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53

6. ANS:
$\sum_{n=1}^{10} 2n$

PTS: 1 DIF: L2 REF: 11-4 Arithmetic Series
OBJ: 11-4.2 Using Summation Notation
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP J.1.7 TOP: 11-4 Example 3
KEY: arithmetic series | summation notation

7. ANS:
1.11

PTS: 1 DIF: L2 REF: 11-5 Geometric Series
OBJ: 11-5.2 Evaluating an Infinite Geometric Series
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP I.1.2 | ADP J.1.7 | ADP J.5.6
TOP: 11-5 Example 4
KEY: evaluating a series | geometric series | infinite geometric series | arithmetic mean

8. ANS:
–338

PTS: 1 DIF: L2 REF: 11-2 Arithmetic Sequences
OBJ: 11-2.1 Identifying and Generating Arithmetic Sequences
NAT: NAEP A1a | CAT5.LV21/22.48 | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.53 | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.I | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.10 | TV.LV21/22.11 | TV.LV21/22.49 | TV.LV21/22.52 | TV.LVALG.53 | TV.LVALG.54
STA: NC A2.1.08 TOP: 11-2 Example 2
KEY: arithmetic sequence | common difference | explicit formula | sequence

9. ANS:
a. Divide the previous term by –2.
b. $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$

PTS: 1 DIF: L2 REF: 11-1 Mathematical Patterns
OBJ: 11-1.1 Identifying Mathematical Patterns
NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 STA: NC A2.1.08
TOP: 11-1 Example 1 KEY: sequence | pattern | multi-part question

10. ANS:
1, 0, 4, -12, 52

PTS: 1 DIF: L2 REF: 11-1 Mathematical Patterns
OBJ: 11-1.2 Using Formulas to Generate Mathematical Patterns
NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP |

S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 TOP: 11-1 Example 4
KEY: explicit formula | recursive formula | sequence | pattern
MSC: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53

11. ANS:
1250

PTS: 1 DIF: L1 REF: 8-6 Geometric Sequences
OBJ: 8-6.1 Geometric Sequences STA: NC 1.02 TOP: 8-6 Example 1
KEY: geometric sequence | common ratio
MSC: NAEP A1a | NAEP A1i | CAT5.LV19.47 | CAT5.LV19.52 | CAT5.LV19.54 | IT.LV15.CP | IT.LV15.DI | IT.LV15.I | S9.TSK1.NS | S10.TSK1.NS | TV.LV19.10 | TV.LV19.16 | TV.LV19.17 | TV.LV19.52 | TV.LVALG.53

12. ANS:
495

PTS: 1 DIF: L2 REF: 11-4 Arithmetic Series
OBJ: 11-4.1 Writing and Evaluating Arithmetic Series
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP J.1.7 TOP: 11-4 Example 2
KEY: arithmetic sequence | arithmetic series | sequence | evaluating a series

13. ANS:
a. $-4 + \left(-\frac{4}{3}\right) + \left(-\frac{4}{9}\right) + \left(-\frac{4}{27}\right)$
b. converge
c. –6

PTS: 1 DIF: L4 REF: 11-5 Geometric Series
OBJ: 11-5.2 Evaluating an Infinite Geometric Series
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP I.1.2 | ADP J.1.7 | ADP J.5.6
TOP: 11-5 Example 4
KEY: common ratio | convergent series | divergent series | evaluating a series | geometric series | infinite geometric series | multi-part question | summation notation

14. ANS:
a. –12, –19
b. $a_n = -0.5n^2 - 0.5n + 9$
c. $a_n = a_{n-1} - n$+where+$a_1 = 8$

PTS: 1 DIF: L4 REF: 11-1 Mathematical Patterns
OBJ: 11-1.2 Using Formulas to Generate Mathematical Patterns
NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 STA: NC A2.1.08
TOP: 11-1 Example 4

KEY: explicit formula | multi-part question | recursive formula | sequence | pattern

15. ANS:
$a_n = 1(.5)^{n-1}$; .0625, .03125

PTS: 1 DIF: L1 REF: 5-6 Describing Number Patterns
OBJ: 5-6.1 Inductive Reasoning and Number Patterns
STA: NAEP A1a | NAEP A1b | CAT5.LV19.47 | CAT5.LV19.52 | CAT5.LV19.54 | IT.LV15.CP | IT.LV15.DI | S9.TSK1.NS | S9.TSK1.PRA | S10.TSK1.NS | S10.TSK1.PRA | TV.LV19.10 | TV.LV19.16 | TV.LV19.17 | TV.LVALG.53 TOP: 5-6 Example 1
KEY: inductive reasoning | conjecture | geometric sequence
MSC: NAEP A1a | NAEP A1b | CAT5.LV19.47 | CAT5.LV19.52 | CAT5.LV19.54 | IT.LV15.CP | IT.LV15.DI | S9.TSK1.NS | S9.TSK1.PRA | S10.TSK1.NS | S10.TSK1.PRA | TV.LV19.10 | TV.LV19.16 | TV.LV19.17 | TV.LVALG.53

16. ANS:
Add 2; 21, 23, 25.

PTS: 1 DIF: L2 REF: 11-1 Mathematical Patterns
OBJ: 11-1.1 Identifying Mathematical Patterns
NAT: NAEP A1a | NAEP A1b | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.DI | IT.LV17/18.DP | S9.TSK3.DSP | S9.TSK3.NS | S10.TSK3.DSP | S10.TSK3.NS | TV.LV21/22.11 | TV.LV21/22.15 | TV.LV21/22.16 | TV.LV21/22.50 | TV.LVALG.53 STA: NC A2.1.08
TOP: 11-1 Example 1 KEY: sequence | pattern

17. ANS:
It diverges; it does not have a sum.

PTS: 1 DIF: L2 REF: 11-5 Geometric Series
OBJ: 11-5.2 Evaluating an Infinite Geometric Series
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP I.1.2 | ADP J.1.7 | ADP J.5.6
TOP: 11-5 Example 3
KEY: geometric series | common ratio | divergent series | convergent series | infinite geometric series

18. ANS: C PTS: 1 DIF: L2 REF: 11-5 Geometric Series
OBJ: 11-5.2 Evaluating an Infinite Geometric Series
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP I.1.2 | ADP J.1.7 | ADP J.5.6
TOP: 11-5 Example 3
KEY: geometric series | common ratio | divergent series | convergent series | infinite geometric series

19. ANS: C PTS: 1 DIF: L2 REF: 11-4 Arithmetic Series
OBJ: 11-4.2 Using Summation Notation
NAT: NAEP A1a | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.52 | IT.LV17/18.AM | IT.LV17/18.CP | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.11 | TV.LV21/22.12 | TV.LV21/22.52 | TV.LVALG.53 | ADP J.1.7 TOP: 11-4 Example 4
KEY: arithmetic series | summation notation | pattern

20. ANS:
a. To write the explicit formula, use the general explicit formula $a_n = a_1 + (n - 1)d$. You can see that the first term, a_1, is –7 and the common difference, *d*, is 1.4. Substitute these values into the formula: $a_n = -7 + (n - 1)1.4$. Simplify the formula to

$a_n = -8.4 + 1.4n$.

b. To write the recursive formula, use the general recursive formula $a_1 =$ a given value, $a_n = a_{n-1} + d$. You can see that the first term is –7 and the common difference *d* is 1.4. Substitute these values into the formula: $a_n = a_{n-1} + 1.4$, where $a_1 = -7$.

c. Answers may vary. Sample: It would be easier to use the explicit formula since it does not depend upon knowing the previous term. You can just substitute 50 for *n* in the explicit formula to find the 50th term.

d. Substitute 103.6 for a_n in the formula $a_n = -8.4 + 1.4n$ and solve for *n*. $103.6 = -8.4 + 1.4n$; 112 = 1.4*n*; *n* = 80. The term 103.6 is the 80th term of the sequence.

PTS: 1 DIF: L4 REF: 11-2 Arithmetic Sequences
OBJ: 11-2.1 Identifying and Generating Arithmetic Sequences
NAT: NAEP A1a | CAT5.LV21/22.48 | CAT5.LV21/22.50 | CAT5.LV21/22.51 | CAT5.LV21/22.53 | IT.LV17/18.AM | IT.LV17/18.CP | IT.LV17/18.I | S9.TSK3.NS | S9.TSK3.PRA | S10.TSK3.NS | S10.TSK3.PRA | TV.LV21/22.10 | TV.LV21/22.11 | TV.LV21/22.49 | TV.LV21/22.52 | TV.LVALG.53 | TV.LVALG.54
STA: NC A2.1.08 TOP: 11-2 Example 2
KEY: arithmetic sequence | common difference | explicit formula | multi-part question | recursive formula | sequence | pattern | writing in math

21. ANS:
jcnalncanscl

PTS: 1